Newton's Laws of Motion

The great meteor crater in Arizona

P sychics and fortune-tellers try to predict the future. Such predictions are rarely confirmed, however. There are simply too many unforeseeable circumstances to allow anyone to predict human affairs reliably. Yet it is sometimes possible to predict the future for mechanical systems. For example, we can predict the future course of a newly observed comet, using Newton's laws of motion. The eighteenth century French scientist Laplace believed that this predictive capacity of Newtonian mechanics could, in principle, be applied even to human events. He wrote:

> If an intellect were to know, for a given instant, all the forces that animate nature and the condition of all the objects that compose her, and were also capable of subjecting these data to analysis, then this intellect would encompass in a single formula the motions of the largest bodies in the universe as well as those of the smallest atom; nothing would be uncertain for this intellect, and the future as well as the past would be present before its eyes.

Although Laplace's belief turned out to be wrong, Newtonian mechanics does have a remarkable predictive capacity, as we shall see in this chapter.

In the three preceding chapters we described motion, using the concepts of velocity and acceleration. However, we have not yet discussed how the motion of a body results from forces acting on the body. In this chapter we shall begin our study of **dynamics,** that part of mechanics that relates the motion of a body to forces exerted on the body by its surroundings. We shall use Newton's laws of motion, together with several force laws, to describe and explain the connection between forces and motion.

4-1   Classical Mechanics

In 1687 Isaac Newton, whose life is described at the end of Chapter 6, published his great work *Philosophiae Naturalis Principia Mathematica.* In the *Principia* Newton established a complete conceptual and mathematical system for understanding motion. He formulated three general laws of motion and used them, along with his law of universal gravitational force, to solve the ancient problem of understanding the solar system. Starting from these laws, Newton was able to calculate planetary orbits precisely. He was also able to explain the behavior of comets and of ocean tides. Today, more than 300 years later, Newton's system of mechanics, called "classical mechanics," is still used to describe that part of nature most accessible to human observation.* Newton's laws are applied to an enormous variety of physical systems. For example, they are used to determine internal forces and stresses in the design of rigid structures; they are used to study the forces acting on and within the human body under various conditions; and they are used to calculate the engine thrust necessary to send a spacecraft to a given destination.

There are two general kinds of problems encountered in classical mechanics:

1 Given the acceleration of a particle, find the forces exerted on the particle by its physical environment. For example, determine the force of air resistance on a parachutist accelerating toward the earth at a given rate.

2 Given a particle's initial position and initial velocity and the forces exerted on it by its physical environment, determine the particle's subsequent motion. For example, given the location and velocity of a comet relative to the sun, determine the comet's position and velocity at any time in the future.

4-2   Force

As a first step in developing the concept, think of force as either a push or a pull exerted by one body on another. Historically the force concept developed from human pushes and pulls and the accompanying feeling of muscular exertion.

Anytime one body exerts a force on a second body, the body exerting the force also experiences a force, called a "reaction force" (Fig. 4-1).

Fig. 4-1   Touching and being touched —action and reaction.

*Only in the twentieth century have Newton's laws failed in their ability to describe physical systems and then only in situations remote from everyday experience, as when an object is moving at nearly the speed of light or when the system is of atomic or subatomic dimensions. We shall study these domains of "modern physics" in Chapters 27 to 30. There we shall find that the laws of classical physics are superseded by the more general laws of relativity and quantum physics. However, we do not need to introduce the more difficult methods of modern physics into the solution of problems that can be successfully solved using classical physics. Furthermore, a thorough grounding in classical mechanics is essential to an understanding of modern physics.

The mutual **interaction** between two bodies is illustrated in Fig. 4-2 for several systems. Certain forces act only when the two bodies are touching, as in examples a, b, c, and f in Fig. 4-2. These are called **contact forces.** There are other forces, however, that act even when the interacting bodies are not touching. This **action at a distance** is easy to observe in the case of two permanent magnets (example d). The gravitational force is another example of a noncontact force. Near the surface of the earth, a force acts on any body, pulling it toward the center of the earth (example e). The body's weight is a measure of this attractive force.

Although contact is not necessary for there to be forces acting between two bodies, the strength of the interaction generally depends on how close to each other the two bodies are. Thus magnets must be fairly close to each other if they are to experience an observable mutual force, and a body must be somewhere in the vicinity of the earth to experience fully the earth's gravitational pull.
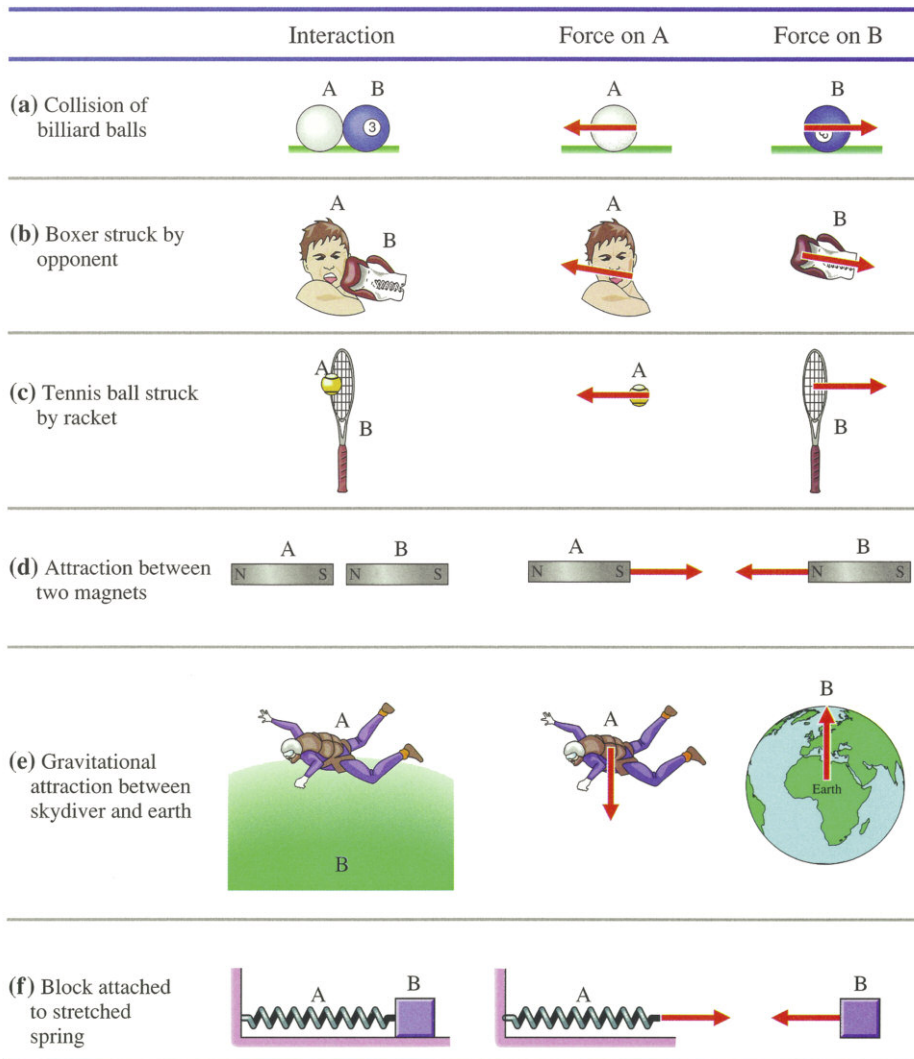
	Interaction	Force on A	Force on B
(a) Collision of billiard balls			
(b) Boxer struck by opponent			
(c) Tennis ball struck by racket			
(d) Attraction between two magnets			
(e) Gravitational attraction between skydiver and earth			
(f) Block attached to stretched spring			

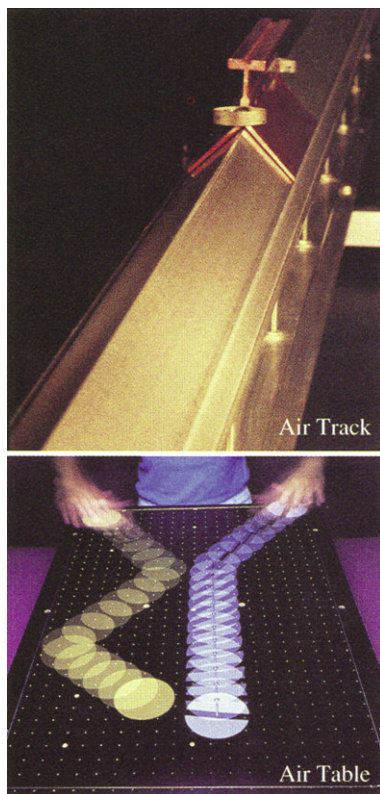**Fig. 4-2** Forces between interacting bodies.

Fig. 4-3   An air track and an air table have surfaces with hundreds of tiny holes bored in them. Air is blown out through the holes, thereby allowing the "cars" to ride on a nearly frictionless cushion of air. Games such as air hockey utilize the same principle.

4-3   Newton's First Law

Newton's Statement

Newton's first law of motion states that *"Every body continues in its state of rest, or of uniform motion in a straight line, except when it is compelled to change that state by forces impressed upon it."* The tendency of a body to maintain its state of rest or of uniform motion in a straight line is called **inertia,** and the first law is sometimes called the **law of inertia.**

If a body either remains at rest or moves uniformly in a straight line, the body's velocity is constant and its acceleration is therefore zero. Thus another way of stating the first law is that: **a body will have zero acceleration if no forces act upon it.**

The first law implies that the effect of a force is to accelerate a body—to change its state of motion. This implication makes more precise our original notion of force as a push or a pull.

Galileo and Aristotle

The first law was partially formulated by Galileo when he was studying objects given an initial velocity on a smooth horizontal plane. Galileo observed that the smoother the surface, the farther an object travels before coming to rest. He concluded that, in the absence of friction, an object would travel forever, no force being necessary to maintain its motion.* Galileo's ideas were in sharp contrast to those of Aristotle, who believed that motion could not exist without the application of force. Aristotle's belief was doubtless derived from common experience, where friction is a factor and where an applied force is necessary to maintain motion by balancing the frictional force. For example, if you want to slide a book along the surface of your desk, you must continuously apply a force to the book in order to cancel the force of friction. Otherwise the book quickly comes to rest.

The air track and air table are devices for producing sliding motion with very little friction (Fig. 4-3), and so they approximate the ideal conditions envisioned by Galileo. So little friction is present on their surfaces that, once an object is given an initial velocity, it continues to move for a considerable time.

Inertial Reference Frames

Is Newton's first law valid for an observer in *any* reference frame? To answer this question, suppose that you are in outer space and observe an isolated body at rest. Another observer, who is accelerating with respect to you, views the same body and observes it to be accelerated. Since the body is isolated, there is nothing around to produce a force on it. Newton's first law is obviously satisfied for you, since both the force on the body and its acceleration equal zero. But for the other observer, Newton's first law is violated because the body appears to be accelerated without any force acting on it.

Whether Newton's first law is satisfied for any given observer depends on the reference frame of the observer. **A reference frame in which Newton's first law (the law of inertia) is satisfied is called an "inertial reference frame."**

Given one inertial reference frame, any other reference frame moving at constant velocity with respect to it is also inertial. In our example, if still another observer comes along, one who is moving at constant velocity relative to you rather than accelerating, she observes the isolated body moving at constant velocity. Newton's first law is satisfied in her reference frame as well as yours.

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*Galileo believed that this ability to travel forever would be true for a perfectly smooth *circular* path around a perfectly spherical earth, rather than for a *straight-line* path. Descartes, a contemporary of Newton, was responsible for recognizing that this principle applies only to linear motion.

Can we name at least one physical reference frame that is inertial? Since the principle of inertia was formulated on earth, it is reasonable to assume that the earth itself is such an inertial frame. This turns out to be a good approximation in many cases but not exactly correct. The first law works in any reference frame with respect to which distant stars are either at rest or moving at constant velocity. It is a remarkable fact that one of the simplest laws of physics, discovered by observation and experiment on earth, is connected to the most distant matter in the universe. Because of the earth's daily rotation, points on the earth experience acceleration with respect to the stars, and so the earth's surface is not a truly inertial reference frame. However, the magnitude of this rotational acceleration is small—only about 0.03 m/s², as shown in Problem 41 of Chapter 3. Therefore, for most practical purposes we can ignore this small acceleration and **take the surface of the earth to be an inertial reference frame.**

It is not only Newton's first law that is valid in any inertial reference frame. It turns out that **all the laws of physics are valid in any inertial reference frame.**

4-4   Mass

Mass is a measure of the inertia of a body; that is, the mass of a body is a measure of the body's resistance to acceleration. Some bodies are harder to accelerate than others. Consider, for example, a bowling ball and a billiard ball, both initially at rest on a billiard table. If you strike the billiard ball with a cue stick, you can easily apply enough force to the ball to give it a significant velocity. The billiard ball is relatively easy to accelerate. Strike the bowling ball with the cue in the same way, however, and it will hardly move. To give the bowling ball the same acceleration you gave the billiard ball would require a much larger force. A bowling ball resists acceleration more than a billiard ball. A bowling ball has more mass than a billiard ball.

How do we quantify the concept of mass? Mass is a fundamental property of matter, just as length is a fundamental property of space (or of matter in space) and time is a fundamental property of existence. We define all these fundamental quantities by defining how we measure them. In the case of length and time, this quantification is familiar and accepted. Length is quantitatively defined when we establish a process for measuring the length of any body. Measurement of a body's length is a comparison between that length and multiples of some standard length, say, the meter. Time is quantified when we establish a process for measuring any time interval with respect to a standard unit of time. Measurement of a time interval is accomplished when we note the readings of a clock at the beginning and end of that interval.

Likewise the concept of mass can be made quantitative by reference to a standard mass. The scientific standard of mass, the standard kilogram, is a cylinder made of a very durable platinum-iridium alloy and kept in a sealed vault in Paris. Copies of this standard are in laboratories all over the world.

The mass of any object can be defined by the following experiment. Place a copy of the standard kilogram (abbreviated kg) on a frictionless surface and apply a force sufficient to give the kilogram an acceleration of 1 m/s² (Fig. 4-4). Next, apply this same force to any other body whose mass you want to determine.* The mass of the body is defined to be the inverse of the acceleration the body experiences under the action of this force. For example, if a body experiences an acceleration of 2 m/s², it has a mass of 0.5 kg by definition. If another body is accelerated at a rate of $\frac{1}{3}$ m/s² by the same force, it has a mass of 3 kg.
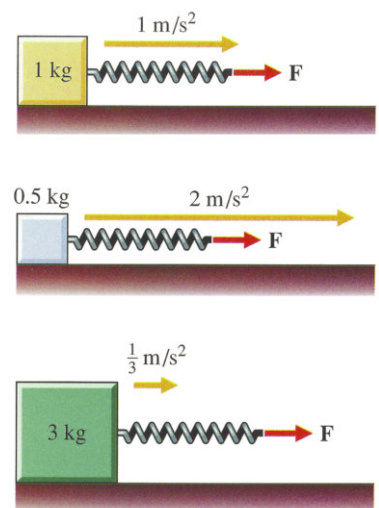

Fig. 4-4   Three bodies of different mass are accelerated by the same force.

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*We can be sure it is the same force by using a spring to apply the force; the same stretching of the spring implies the same force.

Mass is an additive property of matter. If a body of mass m_1 is attached to a body of mass m_2, the mass of the combination is $m_1 + m_2$. For example, if we place a 2 kg mass and a 3 kg mass together on an air track and apply the same force as before, we will observe an acceleration of ⅕ m/s². This means that when we combine the 2 kg and 3 kg masses, we have a total of 5 kg.

We shall show in the next section that the weight of a body is proportional to its mass. This proportionality allows for a much easier method of measuring mass than the method used to define it. As a practical procedure, we can use an equal-arm balance to measure mass (Fig. 4-5). An unknown mass is balanced with multiples or submultiples of the standard mass. Balance is achieved when the forces acting on the two arms of the balance are equal. These forces are equal to the weights of the two masses. Equality of the weights implies equality of the masses.

Because weight and mass are proportional to each other, the two are often confused. It is important to distinguish clearly between them. Mass, a scalar quantity, is a measure of a body's inertia; weight, a vector quantity, is a measure of the earth's gravitational pull on the body.
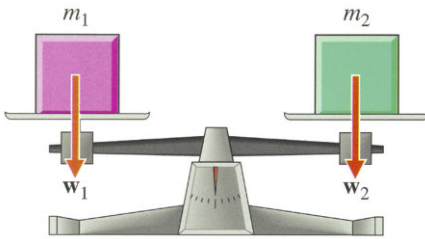

Fig. 4-5   A balance is used to measure mass.

4-5   Newton's Second and Third Laws

Second Law

The acceleration of a particle is determined by the resultant force acting on the particle. According to **Newton's second law** of motion, the acceleration is in the direction of the resultant force $\Sigma\,\mathbf{F}$ (Fig. 4-6), and the magnitude of the resultant force equals the product of mass times acceleration. Using vector notation, the second law is expressed

$$\Sigma\,\mathbf{F} = m\mathbf{a} \tag{4-1}$$

Fig. 4-6   A particle subject to two forces is accelerated in the direction of the resultant force $\Sigma\,\mathbf{F}$.

This vector equation implies that each component of the resultant force equals the mass times the corresponding component of acceleration. For forces in the xy plane, the second law in component form is written

$$\Sigma\,F_x = ma_x \qquad\qquad \Sigma\,F_y = ma_y$$

If we know the acceleration of a particle, we can use the second law to find the resultant force acting on the particle. On the other hand, if we know the forces acting on the particle, we can use the second law to find the particle's acceleration. We can then use the acceleration to predict the future motion of the particle.* When acceleration is the unknown, we may express the second law in the form

$$\mathbf{a} = \frac{\Sigma\,\mathbf{F}}{m} \tag{4-2}$$

or
$$a_x = \frac{\Sigma\,F_x}{m} \qquad\qquad a_y = \frac{\Sigma\,F_y}{m}$$

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*The equations for the position and velocity of a particle undergoing linear motion at constant acceleration are an example of this (Section 2-2).
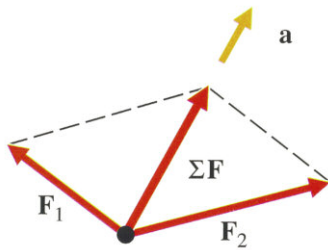
Units

The unit of force is obviously related to units of mass and acceleration by the second law ($\Sigma\,\mathbf{F} = m\mathbf{a}$). We define the newton, abbreviated N, to be the force that produces an acceleration of 1 m/s^2 when acting on a 1 kg mass. Thus

$$1\ \text{N} = 1\ \text{kg-m/s}^2 \tag{4-3}$$

The dyne is the force necessary to accelerate a 1 gram mass at the rate of 1 cm/s^2:

$$
\begin{aligned}
1\ \text{dyne} &= 1\ \text{g-cm/s}^2 = (10^{-3}\ \text{kg})(10^{-2}\ \text{m})/\text{s}^2 \\
&= 10^{-5}\ \text{kg-m/s}^2 \\
&= 10^{-5}\ \text{N}
\end{aligned}
$$

The pound is the unit of force in the British system. Although the pound may be defined independently, it is perhaps simplest to relate it to the newton:

$$1\ \text{lb} = 4.45\ \text{N} \tag{4-4}$$

The unit of mass in the British system is the slug. Since the British unit of acceleration is ft/s^2, we may write

$$1\ \text{slug} = (1\ \text{lb})/(1\ \text{ft/s}^2)$$

or

$$1\ \text{slug} = (4.45\ \text{N})/(0.305\ \text{m/s}^2) = 14.7\ \text{kg}$$

Table 4-1   **Systems of units for force and mass**

System of units	Mass	Acceleration	Force
SI	kilogram	m/s^2	N = kg-m/s^2
cgs	gram	cm/s^2	dyne = g-cm/s^2
			(1 dyne = 10^{-5} N)
British	slug	ft/s^2	lb = slug-ft/s^2
			(1 lb = 4.45 N)

EXAMPLE 1   Computing the Force to Accelerate a Body

Find the force that must be exerted on a 0.500 kg air-track car to give it an acceleration of 3.00 m/s^2.

SOLUTION   According to Newton's second law, the resultant force equals the product of the car's mass and its acceleration:

$$\Sigma\,\mathbf{F} = m\mathbf{a}$$

If we choose the x-axis along the track, we have only an x component of acceleration. Denoting the single horizontal force by $\mathbf{F}$, we find its x component:

$$F_x = ma_x = (0.500\ \text{kg})(3.00\ \text{m/s}^2) = 1.50\ \text{N}$$

Third Law

Newton's third law of motion states that forces result from the mutual interaction of bodies and therefore always occur in pairs, as in Fig. 4-2. The third law states further that **these forces are always equal to each other in magnitude and opposite in direction.** Notice that this last statement does not mean that the forces cancel, since they do not act on the same body. The two forces involved in the third law *always* act on two different bodies. Failure to recognize this point is a common source of error in problem solving.
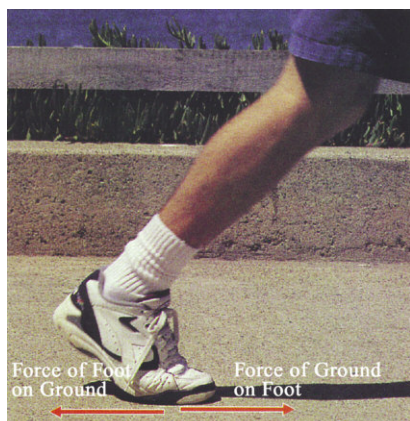
(a)
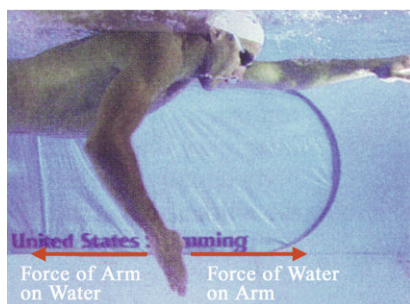
(b)

Fig. 4-7   Using Newton's third law to produce human motion.

The forces occurring in any interaction are often referred to as action and reaction forces. This terminology should not be misinterpreted. Neither force occurs before the other. Either force may be called the action force; the other is then called the reaction force. Action-reaction forces are shown in Fig. 4-2 for several systems.

The third law does not imply that the effect of the two forces will be the same. For example, when a rifle fires a bullet, the forces on the bullet and the rifle have equal magnitude, but the bullet, because of its much smaller mass, experiences a much greater acceleration than the rifle. Or when one boxer punches another in the face, the forces on the face and the fist are equal in magnitude, but the effects of the two forces are quite different.

Newton's third law is utilized in locomotion. For example, in walking you move forward by pushing one foot backward against the floor. The reaction force of the floor on your foot produces the forward acceleration of your body (Fig. 4-7a). In swimming, forward motion is provided primarily by your arms, which push the water backwards, thereby producing a reaction force of the water on your arms in the forward direction (Fig. 4-7b). The flight of birds is also based on this principle.*

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*In analyzing the flight of birds, Leonardo da Vinci (1452–1519) recognized that when a bird's wings thrust against the air, the air pushes back on the wings and thereby supports the bird. But Leonardo's anticipation of Newton's third law as well as his other scientific discoveries had no influence on the development of science because they were unknown until hundreds of years later. Leonardo was so concerned about keeping his discoveries secret that he wrote in a mirror-image code, so that his words could be read only when seen in a mirror.

EXAMPLE 2   Pushing on a Wall

A standing person pushes against a wall with a horizontal force (Fig. 4-8). (a) Why doesn't the section of wall in contact with his hand move? (b) According to the third law, the person's horizontal push on the wall is accompanied by a reaction force on the person. Why doesn't he move away from the wall as a result of this reaction force?

SOLUTION   (a) The section of wall in contact with the hand does not move in response to the applied force because other forces are exerted on it by the other parts of the wall in contact with that section.* Since the wall doesn't move, the resultant of all forces must be zero, according to Newton's second law ($\Sigma \mathbf{F} = m\mathbf{a} = \mathbf{0}$).

Fig. 4-8

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*Actually there is a very slight movement of the surface when it is first pushed. As soon as the surface is slightly deformed, the surrounding parts of the wall begin to create a force opposing that exerted by the hand. If the wall's surface is soft (for example, cork) the deformation is readily observable.

Continued.

EXAMPLE 2—cont'd

(b) The wall certainly exerts an outward force on the hand (Fig. 4-9). If this force were unbalanced, the person would move away from the wall. Since the person is standing at rest, Newton's second law implies that the sum of the forces acting on the person must be zero. So there must be another force acting on the person, one that cancels the outward force of the wall. This other force on the person cannot be the reaction force to the wall's outward push. Remember action-reaction forces always act on different bodies.

The other force acting on the person is provided by the interaction between the feet and the floor. The feet must push out against the floor so that the floor will push back against the feet. As illustrated in Fig. 4-9, this pushing in opposite directions by the wall and floor produces a resultant force of zero. (If the person were on roller skates, $\mathbf{F}_2$ would be smaller than $\mathbf{F}_1$ and the person would move to the right.)
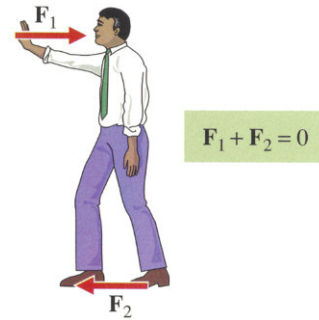
Fig. 4-9

EXAMPLE 3   Finding the Acceleration of a Body

Three astronauts, each of mass 70.0 kg, "float" in an orbiting space station and simultaneously exert forces on a block having a mass of 20.0 kg, as indicated in Fig. 4-10a. (a) Find the x and y components of the block's acceleration. (b) Find the instantaneous acceleration of the astronaut exerting the force $\mathbf{F}_1$.

SOLUTION   **(a)** We apply the component form of Newton's second law to the block, in order to find a_x and a_y:
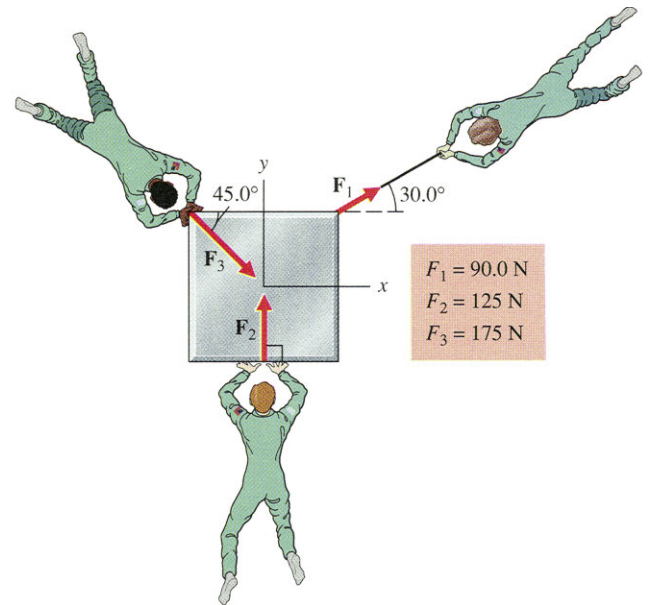
$$a_x = \frac{\Sigma F_x}{m}$$

From the figure, we find the x component of each force and then substitute into our acceleration equation:

$$a_x = \frac{F_{1x} + F_{2x} + F_{3x}}{m}$$

$$= \frac{(90.0 \text{ N})(\cos 30.0°) + 0 + (175 \text{ N})(\cos 45.0°)}{20.0 \text{ kg}}$$

$$= 10.1 \text{ m/s}^2$$

We obtain a_y in the same manner:

$$a_y = \frac{\Sigma F_y}{m} = \frac{F_{1y} + F_{2y} + F_{3y}}{m}$$

$$= \frac{(90.0 \text{ N})(\sin 30.0°) + 125 \text{ N} - (175 \text{ N})(\sin 45.0°)}{20.0 \text{ kg}}$$

$$= 2.31 \text{ m/s}^2$$

| $F_1 = 90.0$ N |
| $F_2 = 125$ N |
| $F_3 = 175$ N |

(a)

Fig. 4-10

Continued.

EXAMPLE 3—cont'd

(b) If we know all the forces acting on the astronaut exerting force $\mathbf{F}_1$, we can apply Newton's second law to find her acceleration. The only force acting on her is the reaction force to the force $\mathbf{F}_1$ she exerts on the block. According to Newton's third law, this reaction force $\mathbf{F}_1'$ is the negative of the force $\mathbf{F}_1$:

$$\mathbf{F}_1' = -\mathbf{F}_1$$

The force $\mathbf{F}_1'$ has the same magnitude as $\mathbf{F}_1$ and is directed 30.0° below the negative x-axis, as shown in Fig. 4-10b. To find the astronaut's acceleration $\mathbf{a}_1'$, we apply Newton's second law:

$$\mathbf{a}_1' = \frac{\Sigma \mathbf{F}}{m} = \frac{\mathbf{F}_1'}{m}$$

This vector equation implies that the astronaut's acceleration is in the same direction as the force $\mathbf{F}_1'$ and has magnitude equal to the magnitude of that force divided by the mass:

$$a_1' = \frac{F_1'}{m} = \frac{90.0 \text{ N}}{70.0 \text{ kg}}$$
$$= 1.29 \text{ m/s}^2$$

$F_1' = F_1 = 90.0 \text{ N}$

(b)

Fig. 4-10, cont'd.

Fig. 4-11   A freely falling body of mass m experiences acceleration **g**.
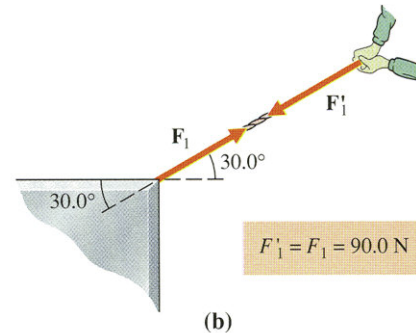
4-6   Force Laws

A force law relates the force on a body to the body's surroundings. In this section we shall discuss several important force laws that will be useful in applying Newton's laws.

Weight on Earth

Perhaps the simplest of all force laws is the gravitational force law for a body of mass m near the surface of the earth. We can find an expression for this force by considering a body of mass m that is falling freely and experiencing negligible air resistance (Fig. 4-11). According to Newton's second law, the resultant force acting on any body equals the product of its mass and acceleration:

$$\Sigma \mathbf{F} = m\mathbf{a}$$

The falling body is subject only to the earth's gravitational force, which we refer to as the body's weight (denoted by **w**); thus the resultant force equals the weight ($\Sigma \mathbf{F} = \mathbf{w}$). We know from experiment that, in the absence of air resistance, all freely falling bodies near the earth's surface experience the same acceleration $\mathbf{a} = \mathbf{g}$, as discussed in Section 2-3. Substituting the resultant force and acceleration into Newton's second law, we obtain an expression for the weight of a body of mass m on earth:

$$\mathbf{w} = m\mathbf{g} \qquad (4-5)$$

Although we have derived this equation for a falling body, we may apply it quite generally to any body on or near the earth's surface. The gravitational force arises from the mutual interaction of the earth and the body. Whenever a body is close to the earth's surface, the body experiences a downward force **w**, equal to the product of its mass m and gravitational acceleration **g**. The same force acts *irrespective of the body's motion or of the presence of other forces.* This is our first example of a force law. It allows us to compute the gravitational force on a body (in other words, the body's weight), given its physical environment (on or near the surface of the earth). We shall use this force law frequently in solving problems.

EXAMPLE 4   Forces on a Man

Find the forces acting on a standing man whose mass is 90.0 kg.

SOLUTION   According to Eq. 4-5, the man experiences a force **w** in the downward direction (the direction of **g**), and the magnitude of this force is

$$w = mg = (90.0 \text{ kg})(9.80 \text{ m/s}^2) = 882 \text{ N}$$

or

$$w = 882 \text{ N}\left(\frac{1.00 \text{ lb}}{4.45 \text{ N}}\right) = 198 \text{ lb}$$

Since the man is standing at rest, his acceleration is zero and so the second law implies that there must be another force to cancel the weight and produce a resultant force equal to zero, as shown in Fig. 4-12. This other force is produced by the contact between his feet and the surface on which he is standing. We denote this surface force by **S** and use the second law to solve for it:

$$\Sigma \mathbf{F} = m\mathbf{a} = 0$$

$$\mathbf{S} + \mathbf{w} = 0$$

Thus $$\mathbf{S} = -\mathbf{w}$$

This equation says that the forces are oppositely directed and have equal magnitudes:

$$S = w = 882 \text{ N}$$

We could have just as easily solved this problem using Newton's second law in component form. Taking the positive y-axis in the upward direction, we have

$$\Sigma F_y = ma_y = 0$$

or $$S - w = 0$$

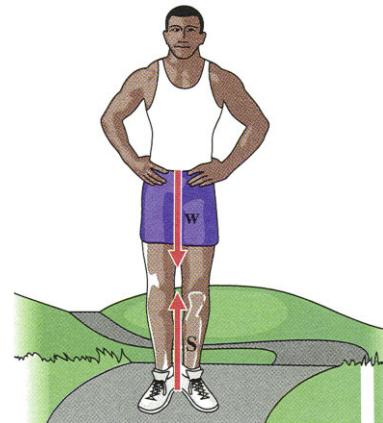Therefore $$S = w = 882 \text{ N}$$

Fig. 4-12

The forces **S** and **w** are equal here because the man is stationary. It is possible for him to increase the force **S** by pushing down on the ground with a force greater than his weight. By Newton's third law, the upward force on his feet will then be greater. There would then be a resultant upward force of magnitude $S - w$, and the man would accelerate upward. In other words, by pushing on the ground with a force greater than his weight, the man can jump.

Variation of Weight on Earth

The value of g varies slightly from point to point on earth. (The variation arises from several factors, to be discussed in Chapter 6.) In particular, g is a function of latitude. For example, at the equator $g = 9.78$ m/s^2, at 40° north latitude $g = 9.80$ m/s^2, and at the North Pole $g = 9.83$ m/s^2. It follows from Eq. 4-5 ($\mathbf{w} = m\mathbf{g}$) that the weight of any object also varies slightly over the surface of the earth. The value of g is less at the equator than at the North Pole by 0.05 m/s^2, which is about 0.5%. Thus the weight of a body is also 0.5% less at the equator than at the North Pole. If you weigh 1000 N (about 225 lb) at the North Pole, you can "lose" about 5 N, or 1 lb, by moving to the equator! You won't be any slimmer, though, because your mass is unchanged.

Fundamental Forces

In light of the apparent diversity of forces one observes in nature, it is a wonderful fact that there are only four fundamental kinds of force:

Gravitational Force   The force of gravity on earth is a special case of the gravitational interaction—that occurring between the earth and a body on or near its surface. We shall see in Chapter 6 that gravitation is a universal phenomenon; an attractive gravitational force acts between *any* two bodies anywhere in the universe.

Electromagnetic Force   Magnetic forces and forces of static electricity are examples of the electromagnetic interaction, which acts between particles having electric charge. Electromagnetic forces are discussed in Chapters 17 to 22.

Nuclear Forces   There are two fundamental nuclear forces: the **strong interaction** and the **weak interaction.** The strong interaction is responsible for the stability of the atomic nucleus, whereas the weak interaction is responsible for the type of radioactivity known as "beta decay." These forces are discussed in Chapter 30.

By the 1970s physicists had discovered that the electromagnetic and weak forces can be regarded as different manifestations of a single force, called the "electroweak force." Some physicists continue to work toward a further unification, developing "grand unified theories," which, if successful, will unify the strong force and the electroweak force. An even more ambitious goal is the unification of all the fundamental forces, including gravity.

The struggle to find unity in the forces of nature is an ongoing one. At one time electricity and magnetism were believed to be unrelated phenomena. As a result of discoveries in the nineteenth century, however, we now know that the force between electric charges and the force between magnets are special examples of a more general electromagnetic interaction. Viewed at the most fundamental level, maybe there really is only one force.

Derived Forces

All forces in nature can in principle be derived from one of the fundamental forces. In particular Eq. 4-5 ($\mathbf{w} = m\mathbf{g}$) can be derived from the general gravitational force law, as we shall show in Chapter 6.

All contact forces arise from electromagnetic interactions between the charged particles in the bodies making contact. For example, the collision of billiard balls, a boxer's punch, the pressure on a body submerged in water, and the frictional force on a car's tires all arise from electromagnetic forces acting between the interacting bodies. Even the forces holding matter together—atom to atom—are electromagnetic in origin.


"WHAT YOU HAVE DONE, GRUNDIG, IS HELP CONTRIBUTE TO A DIS-UNIFIED THEORY."

Fig. 4-13   (*Physics Today* 41:9, Sept 1988.)

Tension

Typically the forces acting between the parts of a solid body are complex, and so it would be very difficult to find a general force law for computing them. The special case of a flexible body, such as a rope or a string, is somewhat simpler. Again it would be difficult to find an expression for the magnitude of the forces acting within the string, since such an expression depends on particular qualities of the string. However, we can say something about the direction of these forces.

The fact that a string is flexible means that it bends when you push on it. In other words, a string cannot transmit a push. It can of course transmit a pull. The shape of the string adjusts itself so that this force acts along the string. Any section of a flexible rope or string exerts a force on any adjacent section. This force, called **"tension," is a pull tangent to the string** (Fig. 4-14).
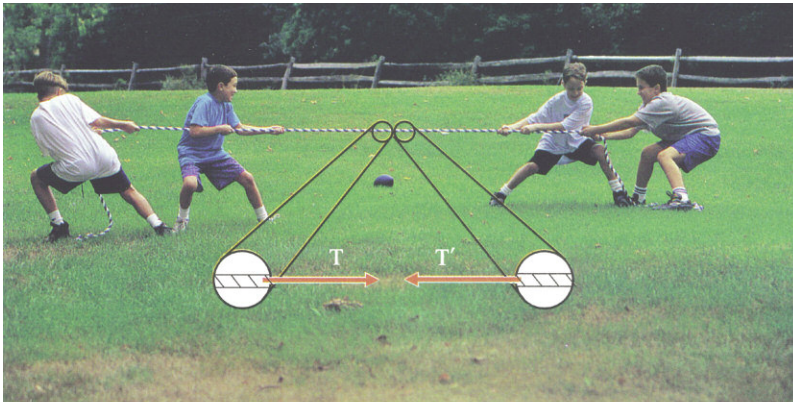
(a)

(b)

Fig. 4-14 **(a)** In a tug-of-war, the rope is under great tension, meaning that there is a large tension force exerted by any section of the rope on an adjacent section. **(b)** A much greater tension is present in the cables supporting the Golden Gate Bridge. At point **P**, the section to the right of **P** exerts a force **T** on the section to the left of **P**. (There is also a reaction force, not shown in the figure.)

The cable consists of 27,572 strands of flexible wire.
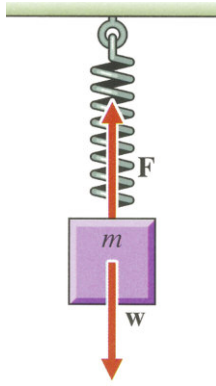
Spring Force

The force exerted by a stretched spring is a particularly simple example of a contact force. When a spring is stretched, some of the adjacent molecules within the spring are pulled slightly farther apart from each other, and an attractive electromagnetic force attempts to pull them back to their original positions. Compression of a spring also produces a force in the spring. In this case, adjacent molecules are pushed together, and it is a repulsive electromagnetic force that is at work, attempting to push the molecules back to their original positions.

When an object hangs vertically at rest from a spring, Newton's second law predicts that the spring exerts a force **F** sufficient to cancel the object's weight (Fig. 4-15). Thus the force exerted by the spring is equal in magnitude to the weight supported. We can experimentally determine a force law for a stretched spring by hanging weights from the spring and measuring the corresponding stretch. When we do so, we find that most springs stretch or compress in direct proportion to the force applied to them, so long as the amount of stretching or compression is not too large. Put another way, the magnitude of the force **F** exerted by the spring is directly proportional to the spring's change in length $\Delta\ell$. This may be expressed

$$F = k\,\Delta\ell$$

where k is called the force constant of the spring. The force constant indicates the stiffness of the spring. The larger the value of k, the stiffer the spring, that is, the larger the force that must be applied to produce a given change in length $\Delta\ell$.

We can express the spring force law in a useful alternative form, a form that indicates the direction as well as the magnitude of the spring force. Consider the force **F** exerted on a block by a horizontal spring, as shown in Fig. 4-16. The origin of the x-axis is chosen at the position of the block for which the spring is relaxed (neither stretched nor compressed). The magnitude of x gives the spring's change in length ($\Delta\ell$) as it is either stretched or compressed. When x is positive, the spring is stretched and exerts a pull to the left, so that F_x is negative. When x is negative, the spring is compressed and exerts a push to the right so that F_x is positive. In either case, the sign of F_x is opposite the sign of x. Both the magnitude and the direction of the spring force are indicated by writing the force law in the form

$$F_x = -kx \qquad\qquad (4\text{-}6)$$
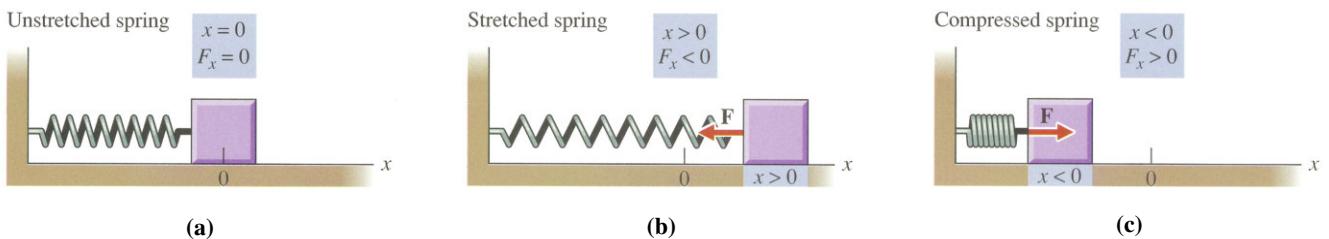

Fig. 4-15   Forces on a mass suspended from a spring.

Fig. 4-16   The force **F** exerted by a spring on a block attached to the spring varies in magnitude and direction, depending on the compression or stretching of the spring.

EXAMPLE 5 Mass on a Spring

Find the instantaneous acceleration of a 1.00 kg mass suspended from a spring of force constant 5.00 N/cm, when the spring is stretched 10.0 cm. The mass is initially at rest.

SOLUTION The forces **F** and **w** acting on the mass are shown in Fig. 4-17. We take the y-axis to be positive in the upward direction. Using Eq. 4-6 with y substituted for x, we obtain the y component of the force exerted on the mass by the spring:

$$F_y = -ky = -(5.00 \text{ N/cm})(-10.0 \text{ cm}) = +50.0 \text{ N}$$

The only other force acting on the mass is its weight **w**, which acts along the negative y-axis and has magnitude given by Eq. 4-5:

$$w = mg = (1.00 \text{ kg})(9.80 \text{ m/s}^2) = 9.80 \text{ N}$$

The spring force exceeds the weight. Therefore the second law predicts an upward acceleration:

$$a_y = \frac{\Sigma F_y}{m} = \frac{50.0 \text{ N} - 9.80 \text{ N}}{1.00 \text{ kg}}$$

$$= 40.2 \text{ m/s}^2$$

The acceleration we have calculated is instantaneous, corresponding to a particular value of y. The acceleration is not a constant. As the mass moves, the length of the spring changes, and therefore the force the spring exerts also changes. The changing force produces changing acceleration.
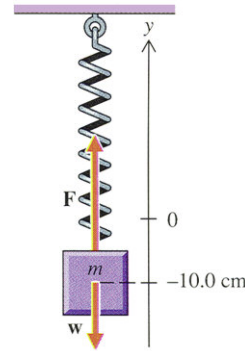
Fig. 4-17

4-7 The Concept of Force

Force is a subtle physical concept, one that developed over hundreds of years. It is therefore not surprising that understanding the precise nature of this concept requires some careful thought. We began our discussion of force in this chapter with the simple qualitative concept of a push or a pull. With our discussion of Newton's first and second laws of motion, we arrived at a refinement of the force concept as that which tends to produce acceleration. It is sometimes stated that force is "defined" by Newton's second law to be mass times acceleration. The difficulty with this kind of statement is that it leaves the impression that the second law is merely a definition* and therefore that it says nothing substantive about nature. But implicit in the second law is the idea that there are force laws—equations for computing the force on a body from knowledge of its physical environment.

The essential physical fact is that the interaction between an object and its physical environment can produce an acceleration of the object. If there were only one kind of force in nature, we could express the relationship between the acceleration and the environment directly and eliminate the concept of force. As it is, there are different kinds of interactions in nature and many different force laws.

Thus force is a useful intermediate concept—a unifying element in the logical structure of physics. The force concept is a way of relating the motion of a body to the body's surroundings. We think of a body experiencing "forces" produced by other bodies in the surroundings. Each force is a vector whose magnitude and direction can sometimes be computed from one of a number of force laws. When all the forces acting on the body are known, the second law can be used to find its acceleration.

*However, the second law is used to define the *unit* of force (1 N = 1 kg-m/s²).

4-8   Applications of Newton's Laws of Motion

As stated at the beginning of this chapter, there are two kinds of problems in classical mechanics: (a) to find unknown forces acting on a body, given the body's acceleration, and (b) to predict the future motion of a body, given the body's initial position and velocity and the forces acting on it. For either kind of problem, we use Newton's second law ($\Sigma\, \mathbf{F} = m\mathbf{a}$). The following general strategy is useful for solving such problems:

1. **Choose a body** to which you will apply Newton's second law and isolate that body by drawing a diagram of it free of its physical surroundings. The body chosen is called a **free body,** and the diagram, which will include forces as described in step 2, is called a **free-body diagram.**

   The free body may be a whole body, part of a body, or a collection of bodies. We may apply the second law to any system, as long as the acceleration **a** is the same for all parts of that system. In this case, the system behaves as a particle and Newton's second law is valid (see Problem 52). This condition on **a** is satisfied for any body at rest, such as in Ex. 4, where **a = 0** for all parts of the human body, or for any rigid body moving without rotating,* as for the mass on the spring in Ex. 5.

2. **Identify all the forces** exerted *on* the free body by objects in the surroundings, and draw these forces in the free-body diagram. Any object that is in contact with the free body will exert a force on it. In addition, there may be various noncontact forces: gravitational, electric, or magnetic. In this chapter the only noncontact force we shall need to consider is the weight of the object.

   Do not include in the free-body diagram the forces exerted *by* the free body on the surroundings. Include only forces acting *on* the body.

   Nor should you include forces acting *between* parts of the free body. Thus in Ex. 4 we considered the standing person as a particle, ignoring the human body's internal structure. Of course the body has various parts, each of which exerts forces on other parts. These are internal forces, however, and **only external forces should be included in the free-body diagram because only these forces determine the free body's acceleration.**

   At times we may be interested in computing forces that are normally regarded as internal forces. We can compute such forces if we make an appropriate choice of the body to which we apply Newton's second law, so that these forces are external to the body. For example, we may find the tension in a rope by choosing a section of the rope as the free body, so that the tension is an external force.

3. Choose an inertial reference frame with convenient coordinate axes, **apply force laws,** and **apply Newton's second law** in component form. This step may require resolving force vectors into their components along the coordinate axes.

   If the choice of the free body was a good one, there will be enough information to solve for the unknowns in the problem. Some problems may require the analysis of two or more related free-body diagrams.

---

*The particles of a rotating body do not all experience the same acceleration. However, in the next chapter we shall find that Newton's second law may still be used to describe the motion of a certain point—the center of mass of the body.

EXAMPLE 6   Finding the Tension in a Cable

A block of marble whose weight is 2.00×10^4 N is suspended from a cable supported by a crane (Fig. 4-18a). The cable's weight is 4.00×10^2 N. (a) Find the tension in the top and bottom of the cable when the block and cable are both at rest. (b) Find the tension in the top and bottom of the cable when the block is accelerating downward at the rate of 2.50 m/s².

SOLUTION   **(a)** To find the tension in the top of the cable, choose as a free body the block and cable. Such a choice makes the tension $\mathbf{T}_1$ an external force. This tension force is the only contact force acting on the system. The only other external forces are the weight of the block $\mathbf{w}_b$ and the weight of the cable $\mathbf{w}_c$. The three external forces are shown in the free-body diagram of Fig. 4-18b.

Since the system is unaccelerated, we know from Newton's second law that the vector sum of the external forces equals zero. We choose our coordinate axes as indicated in Fig. 4-18b so that all forces lie along the y-axis and we need only apply the equation

$$\Sigma F_y = 0$$

From our free-body diagram, we see that $\mathbf{T}_1$ acts along the positive y-axis and $\mathbf{w}_c$ and $\mathbf{w}_b$ act along the negative y-axis. Thus

$$T_1 - w_c - w_b = 0$$

Solving for T_1, we obtain

$$T_1 = w_c + w_b = 4.00 \times 10^2 \text{ N} + 2.00 \times 10^4 \text{ N}$$

$$= 2.04 \times 10^4 \text{ N}$$

Next we find the tension in the bottom of the cable by choosing the block and a small section of cable at the bottom as the free body (Fig. 4-18c). The tension $\mathbf{T}_2$ and the weight $\mathbf{w}_b$ are the only external forces. Again we apply the second law:

$$\Sigma F_y = 0$$

$$T_2 - w_b = 0$$

$$T_2 = w_b = 2.00 \times 10^4 \text{ N}$$

The values for T_1 and T_2 are not surprising. Tension T_1 in the top of the cable balances the combined weight of the cable and block, whereas tension T_2 in the bottom of the cable balances the weight of the block alone.

(b) Here the block and cable have an acceleration $a_y = -2.50$ m/s². We shall apply Newton's second law and shall therefore need to find the masses of the block and the cable, m_b and m_c, using the force law $w = mg$:

$$m_b = \frac{w_b}{g} = \frac{2.00 \times 10^4 \text{ N}}{9.80 \text{ m/s}^2}$$

$$= 2040 \text{ kg}$$

$$m_c = \frac{w_c}{g} = \frac{4.00 \times 10^2 \text{ N}}{9.80 \text{ m/s}^2}$$

$$= 40.8 \text{ kg}$$

We again use the free body shown in Fig. 4-18b to solve for T_1, applying Newton's second law:

$$\Sigma F_y = ma_y$$

$$T_1 - w_c - w_b = (m_c + m_b)a_y$$

$$T_1 = w_c + w_b + (m_c + m_b)a_y$$

$$= 4.00 \times 10^2 \text{ N} + 2.00 \times 10^4 \text{ N} +$$
$$(2040 \text{ kg} + 40.8 \text{ kg})(-2.50 \text{ m/s}^2)$$

$$= 1.52 \times 10^4 \text{ N}$$

And using Fig. 4-18c, we find the tension T_2:

$$\Sigma F_y = ma_y$$

$$T_2 - w_b = m_b a_y$$

$$T_2 = w_b + m_b a_y$$

$$= 2.00 \times 10^4 \text{ N} + (2040 \text{ kg})(-2.50 \text{ m/s}^2)$$

$$= 1.49 \times 10^4 \text{ N}$$

Notice that the tensions T_1 and T_2 are now less than the weights supported. The reason is that the weights are accelerating downward. If the acceleration were equal to $\mathbf{g}$, the tension forces would be zero.
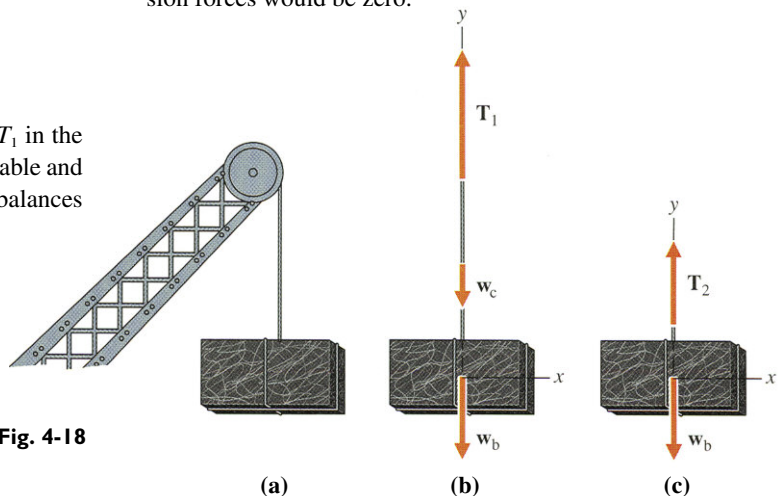

Fig. 4-18

(a)   (b)   (c)

It is a good approximation to ignore the mass of a cable, rope, or string whenever this mass is much less than other masses in a problem. The tension then is transmitted undiminished throughout, a fact that can be seen in the preceding example when we set m_c and w_c equal to zero. Then $T_1 = T_2$ in both parts a and b.

EXAMPLE 7   Forces on a Foot

Find the forces on each foot of a woman standing at rest if her weight of 575 N (129 lb) is evenly distributed between her two feet. Neglect the weight of the foot.

SOLUTION   To solve for the forces exerted on the feet by the supporting surface, we choose the woman as the free body. The weight **w** and two equal contact forces **S** are the only external forces acting on the body (Fig. 4-19a). Applying Newton's second law, we solve for the magnitude of **S**:

$$\Sigma F_y = ma_y = 0$$

$$S + S - w = 0$$

$$S = \frac{w}{2} = \frac{575 \text{ N}}{2} = 288 \text{ N} \text{ (about 65 lb)}$$

We have found the force exerted on either foot by the supporting surface, but this is not the only force acting on the foot. In addition, the leg and upper body exert a downward force on the foot. To solve for this unknown force, we choose the foot alone as the free body and draw our free-body diagram (Fig. 4-19b), with two external forces: **S**, produced by the contact with the supporting surface, and **F**, produced by the contact with the leg and upper body. We neglect the weight of the foot, which is small compared with these other forces. Newton's second law implies that the two forces cancel:

$$\mathbf{F} = -\mathbf{S}$$

$$F = S = 288 \text{ N}$$

(a)   **(b)**
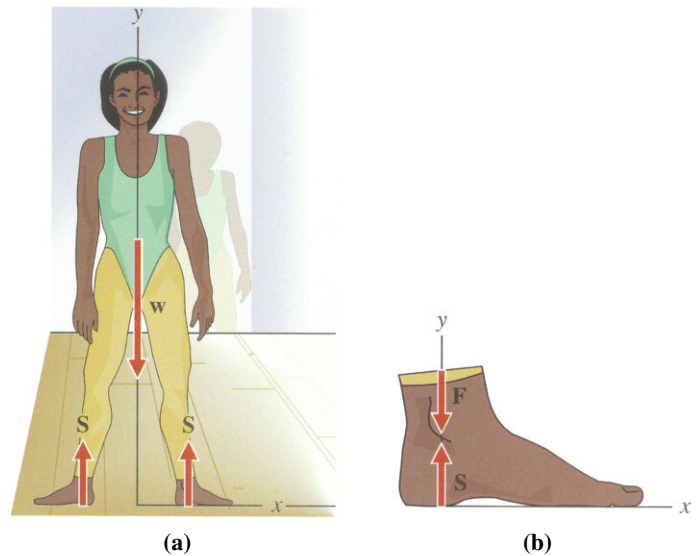
Fig. 4-19

The two forces **F** and **S** are both equal in magnitude to half the weight of the body. In other words, half the weight of the body pushes down on each foot and is supported by the surface. The two opposing forces **F** and **S** produce no acceleration of the foot, but they do cause some compression.

EXAMPLE 8 Forces on Accelerating Blocks

Two blocks are pushed along a frictionless horizontal surface by a constant 6.00 N force (Fig. 4-20). Find the acceleration of each block and the forces on it, given that $m_1 = 1.00$ kg and $m_2 = 2.00$ kg.

SOLUTION We may choose m_1, m_2, or the combination of m_1 and m_2 as a free body, since m_1 and m_2 have a common acceleration. The three free-body diagrams are shown in Fig. 4-21. Notice that there is a contact force $\mathbf{F}_2$ exerted on m_2 by m_1 and a reaction force $\mathbf{F}_2'$ exerted on m_1 by m_2. This contact force must be smaller than the applied force $\mathbf{F}_1$; otherwise there would be no net force to provide for the acceleration of m_1.

In solving this problem, we begin with the free-body diagram of the two-block combination because in the other diagrams there are too many unknowns. There is no motion in the vertical direction, and thus $a_y = 0$ and the surface forces and weights cancel. We apply Newton's second law to the motion along the x-axis and solve for a_x:

$$\Sigma F_x = ma_x$$

$$F_1 = (m_1 + m_2)a_x$$

$$a_x = \frac{F_1}{m_1 + m_2} = \frac{6.00 \text{ N}}{1.00 \text{ kg} + 2.00 \text{ kg}}$$

$$= 2.00 \text{ m/s}^2$$
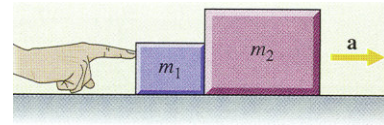
Fig. 4-20

Now that we have found a_x, we may apply Newton's second law to m_2 and solve for the unknown $\mathbf{F}_2$:

$$\Sigma F_x = ma_x$$

$$F_2 = m_2 a_x = (2.00 \text{ kg})(2.00 \text{ m/s}^2)$$

$$= 4.00 \text{ N}$$

We can check this result by applying Newton's second law to m_1 and solving for a_x:

$$a_x = \frac{\Sigma F_x}{m}$$

$$= \frac{F_1 - F_2}{m} = \frac{6.00 \text{ N} - 4.00 \text{ N}}{1.00 \text{ kg}}$$

$$= 2.00 \text{ m/s}^2$$

This, of course, agrees with our previously computed value of a_x.

(a) m_1 (b) m_2 (c) m_1 and m_2
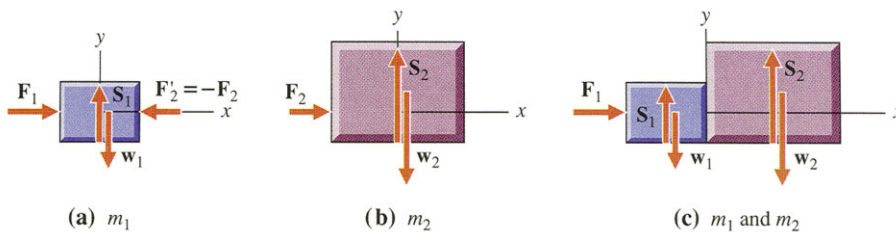
Fig. 4-21

EXAMPLE 9 Tension in Strings Supporting a Weight

A 10.0 N weight is supported at rest by string of negligible mass, as shown in Fig. 4-22a. Find the tension in each string.

SOLUTION The tension throughout the vertical string is obviously just equal to the weight supported—10.0 N. (You can prove this result by choosing the weight and any section of the vertical string as a free body and applying Newton's second law.)

The tension in the other two strings is not so obvious. We must choose a free body for which these forces are external forces and for which there is sufficient information to solve for the forces. In problems such as this, the right choice for the free body may not be apparent. There are many bodies one might choose but from which no information is gained—the ceiling, for example, or a section of one string. The useful free body here is either the knot where the three strings meet or the knot and some section of each string. The three tension forces are all external to the knot; they are shown resolved into vector components in the free-body diagram (Fig. 4-22b).

We already know that the tension T_3 is 10.0 N. We apply Newton's second law in component form to the knot:

$$\Sigma F_x = ma_x = 0$$

$$T_2 \cos 30.0° - T_1 \cos 45.0° = 0$$

This gives us a relationship between the two unknowns T_1 and T_2. A second equation relating T_1 and T_2 is obtained when we equate the sum of the y components of the forces to zero:

$$\Sigma F_y = ma_y = 0$$

$$T_1 \sin 45.0° + T_2 \sin 30.0° - T_3 = 0$$
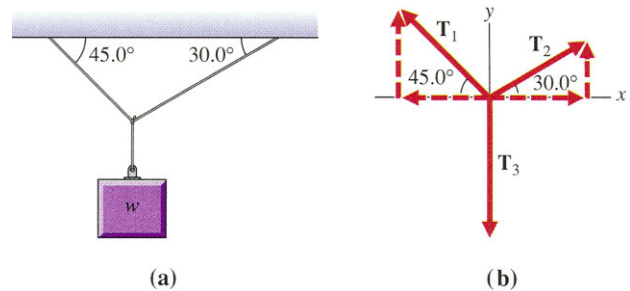
Fig. 4-22

Solving the two linear equations for the two unknowns in terms of T_3 and the angles, we obtain

$$T_1 = \frac{T_3}{\sin 45.0° + \cos 45.0° \tan 30.0°}$$

$$= \frac{10.0 \text{ N}}{\sin 45.0° + \cos 45.0° \tan 30.0°}$$

$$= 8.97 \text{ N}$$

$$T_2 = \frac{T_1 \cos 45.0°}{\cos 30.0°}$$

$$= \frac{(8.97 \text{ N})(\cos 45.0°)}{\cos 30.0°}$$

$$= 7.32 \text{ N}$$

When a flexible rope or string passes over a frictionless pulley of negligible mass, the tension is the same on both sides of the pulley. A frictionless and massless pulley changes the direction of the tension force but leaves the magnitude unchanged (see Problem 45 of Chapter 9).

EXAMPLE 10 Atwood's Machine

Two unequal masses, m_1 and $m_2 > m_1$, are suspended from opposite ends of a rope of negligible mass that passes over and is supported by a frictionless, stationary pulley of negligible mass (Fig. 4-23a). The greater mass m_2 will accelerate downward and the smaller mass m_1 will experience an acceleration of equal magnitude in the upward direction. By adjusting the values of m_1 and m_2, we can make the acceleration as small as we want. (This simple device is called "Atwood's machine.") Find expressions for the magnitude of the acceleration and the tension in the rope as functions of m_1 and m_2.

SOLUTION   First we choose as free bodies the two masses (Fig. 4-23b). The tension force on each mass is the same, and the two accelerations a_1 and a_2 are equal in magnitude and opposite in direction. This follows from the fact that when one mass moves a certain distance upward, the other moves the same distance downward in the same time interval.

We apply Newton's second law to each body:

$$\Sigma F_y = ma_y$$

With our choice of coordinate axes, $a_y = +a$ for m_1 and $a_y = -a$ for m_2. Thus we have

$$T - w_1 = m_1 a$$

$$T - w_2 = m_2(-a)$$

We have two linear equations with two unknowns, T and a. Solving for the unknowns in terms of the masses and weights, we obtain

$$a = \frac{w_2 - w_1}{m_2 + m_1} \qquad T = w_1 + \frac{m_1(w_2 - w_1)}{m_2 + m_1}$$

or, using $w = mg$,

$$a = \frac{m_2 - m_1}{m_2 + m_1} g \qquad T = \frac{2m_1 m_2}{m_2 + m_1} g$$

(a)   **(b)**
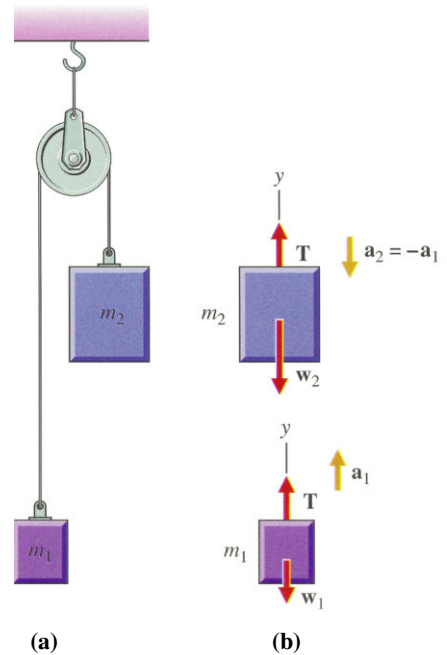
Fig. 4-23

EXAMPLE 11   Instantaneous Force On a Runner's Foot

Fig. 4-24 shows a simplified model of a force platform used in biomechanical research to study the force exerted on the ground by the foot of a running person. Suppose that the platform has a mass of 5.0 kg and each of the four springs has a force constant of 1.0×10^6 N/m.

At some instant, the vertical springs are compressed 0.50 mm. At the same time, each horizontal spring differs from its relaxed length by 0.10 mm, the left spring compressed and the right spring stretched. The platform has a vertical component of acceleration of 5.0 m/s² in the upward direction and a horizontal component of 2.0 m/s² in the backward direction. Find the horizontal and vertical components of force on the platform.

SOLUTION   We choose the platform as the free body and show in the free-body diagram the four forces exerted by the springs—$\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$—the weight of the platform $\mathbf{w}$, and the force exerted by the foot, $\mathbf{F}_5$ (Fig. 4-25). We apply Newton's second law to the motion along the x-axis and solve for the horizontal component of $\mathbf{F}_5$:

$$\Sigma F_x = ma_x$$

$$F_3 + F_4 + F_{5x} = ma_x$$

$$F_{5x} = ma_x - F_3 - F_4$$

The two horizontal spring forces, $\mathbf{F}_3$ and $\mathbf{F}_4$, are of equal magnitude $k \, \Delta \ell$. Substituting into the last equation, we obtain

$$F_{5x} = ma_x - 2k \, \Delta \ell$$
$$= (5.0 \text{ kg})(-2.0 \text{ m/s}^2) - 2(1.0 \times 10^6 \text{ N/m})(1.0 \times 10^{-4} \text{ m})$$
$$= -210 \text{ N}$$

Next we apply the second law to the motion along the y-axis:

$$\Sigma F_y = ma_y$$

$$F_1 + F_2 + F_{5y} - w = ma_y$$

$$F_{5y} = ma_y + w - F_1 - F_2$$

or, using $F_1 = F_2 = k \, \Delta \ell$ and $w = mg$,

$$F_{5y} = ma_y + mg - 2k \, \Delta \ell$$
$$= (5.0 \text{ kg})(+5.0 \text{ m/s}^2) + (5.0 \text{ kg})(9.80 \text{ m/s}^2) -$$
$$\qquad 2(1.0 \times 10^6 \text{ N/m})(5.0 \times 10^{-4} \text{ m})$$
$$= -930 \text{ N}$$

At the instant considered, the foot exerts on the ground a backward force of 210 N (47 lb) and a downward force of 930 N (210 lb). The ground therefore exerts a reaction force of 210 N in the forward direction and 930 N in the upward direction. The graph in Fig. 4-24 indicates that the vertical force on the foot of a 68 kg (150 lb) runner reaches a maximum value
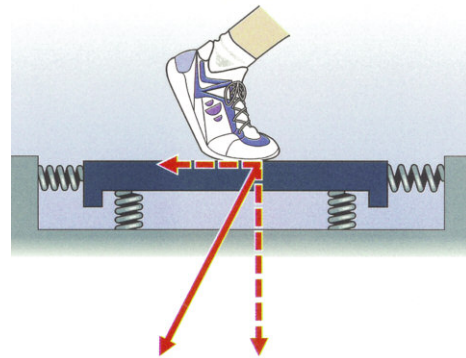
(a)
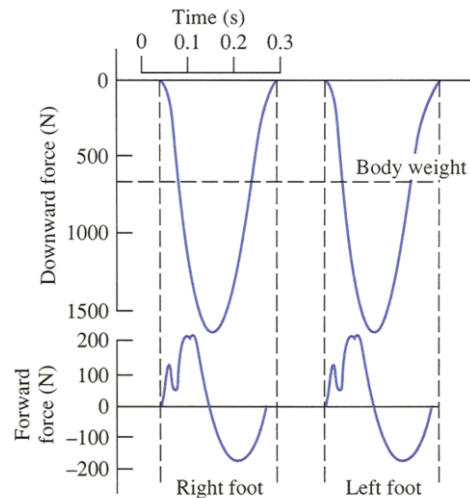
(b)

Fig. 4-24   A force platform and data for a 68 kg man running at 3.5 m/s. (From Alexander R McN: *Biomechanics*, New York, 1975, Halstead Press.)
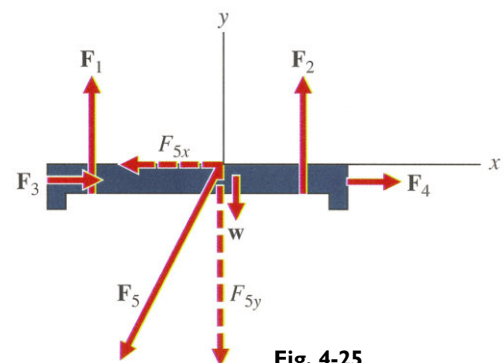

Fig. 4-25

of about 1700 N (380 lb), about 2.5 times the weight of the runner. This suggests why running on a hard surface without proper shoes can so easily lead to injuries. A well-cushioned heel on a running shoe reduces the maximum force on the foot as it hits the surface by lengthening the time of contact and thereby reducing the maximum instantaneous acceleration.

CHAPTER 4 SUMMARY

Mass is a measure of a body's inertia, that is, its tendency to resist acceleration. **Force** is the result of a mutual interaction between two bodies. Forces can sometimes be calculated from force laws. Each force on a body tends to accelerate the body in the direction of the force. This tendency may be opposed by the presence of other forces.

Newton's Laws of Motion

First law If no forces act on a body, the body continues in its state of rest or of uniform motion in a straight line.

Second law The resultant force on a body equals the product of the body's mass and acceleration.

$$\Sigma\, \mathbf{F} = m\mathbf{a}$$

or

$$\Sigma\, F_x = ma_x \qquad \Sigma\, F_y = ma_y$$

Third law Forces occur in action-reaction pairs—two forces equal in magnitude and opposite in direction, acting on two different bodies.

Force Laws

Weight The gravitational force of the earth on a body of mass m near the surface of the earth is its weight $\mathbf{w}$, where

$$\mathbf{w} = m\mathbf{g}$$

Tension The tension in a flexible body, such as a rope or string, is an attractive force between adjacent sections of the rope or string, and tangent to it.

Spring force The magnitude of the force exerted by a spring on an object is related to the change in length $\Delta\ell$ of the spring by the equation

$$F = k\,\Delta\ell$$

where k is the force constant of the spring, a measure of its stiffness. The x component of this force may be expressed as

$$F_x = -kx$$

In applying Newton's second law, any body or combination of bodies may be used as the free body as long as all parts of the free body have the same acceleration $\mathbf{a}$. Then one may draw a free-body diagram, which shows all the external forces acting on the body. These forces determine the acceleration of the body, through the second law. In your analysis you must use an **inertial reference frame,** any reference frame in which Newton's first law is satisfied, that is, in which an isolated body is not accelerated. The laws of physics are valid only in inertial reference frames.

Questions

1 Aristotle had to invent an elaborate process in order to describe the motion of projectiles as forced motion. He argued that an arrow moves through the air by pushing aside the air, which then rushes around to the tail of the arrow and propels it forward. According to Newton, what force is needed to produce the horizontal component of the arrow's velocity?

2 You apply the brakes on your car, stopping suddenly, and are thrown forward. What force is responsible for your forward motion?

3 As viewed from the earth, a body is at rest. The same body is viewed by an observer on an escalator moving at a constant speed of 3 m/s. Are Newton's laws satisfied for this observer?

4 Suppose you are in a completely enclosed compartment in an airplane flying to an unknown destination. The walls are shielded so that you can't detect the earth's magnetic field, and you are not able to observe anything else outside the compartment. You place a ball on the floor, and it remains at rest.
(a) What can you conclude about the velocity of the plane?
(b) Is there any other experiment you could perform to determine the plane's speed or direction of motion? The compartment is a well-equipped physics laboratory.

5 Consider a planet the same size as earth, but one on which a day is much shorter than an earth day. Compared to the surface of the earth, would the surface of the planet be better or worse, as an approximation to an inertial reference frame?

6 Is it possible for an object to move along a curved path without any force acting on it?

7 You are pulling in a fish, using a fishing line that is very close to its breaking point. Should you (a) pull the fish in as quickly as possible or (b) pull the fish in slowly and without jerking the line?

8 If a horse tries to pull a cart, exerting a force on the cart in the forward direction, the cart will exert a backward force on the horse.
 (a) Since these two forces are equal in magnitude and opposite in direction, is it not then impossible for the horse and cart to move?
 (b) If the horse and cart together are considered as the free body, what other body exerts the force necessary to accelerate the free body forward?

9 In a tug of war, the winning team pulls on a rope of negligible mass and drags the losing team across a line. Does the winning team (a) pull harder on the rope than the losing team or (b) push harder on the ground than the losing team?

10 Blocks A and B collide on a frictionless horizontal surface. Block A, of mass 5 kg, experiences an instantaneous acceleration of 10 m/s² to the right, while block B experiences an instantaneous acceleration of 2 m/s² to the left. What is the mass of B?

11 You are an overweight space-age commuter, traveling from planet to planet and so experiencing varying values of g. Which way can you be sure the diet you are following is effective—by measuring (a) your weight on a spring scale, or (b) your mass on a balance?

12 If the earth's pull on a 40 N brick is 10 times as great as its pull on a 4 N book, when both are in free fall, why do they have the same acceleration?

13 An astronaut of mass m is in a spaceship accelerating vertically upward from the earth's surface with acceleration of magnitude a. The contact force exerted on the astronaut has a magnitude given by (a) mg; (b) ma; (c) $m(g + a)$; (d) $m(g - a)$; (e) $m(a - g)$.

14 A skydiver is observed to have a terminal speed of 55 m/s in a prone position and 80 m/s in a vertical position. Which of the following can be concluded from this observation?
 (a) The force of gravity on the skydiver is less in the prone position than in the vertical position.
 (b) The force of air resistance on the skydiver is proportional to the speed of the body.
 (c) The force of air resistance is greater at 55 m/s in the prone position than at 80 m/s in the vertical position.
 (d) The force of air resistance at 55 m/s in the prone position is the same as at 80 m/s in the vertical position.
 (e) None of the above.

15 A heavy blanket hangs from a clothesline. Will the tension in the clothesline be greater if it sags a little or if it sags a lot?

16 In analyzing the forces on a halfback running with a football, a free-body diagram is drawn. If the player's entire body is the chosen free body, which of the following forces should not be drawn in the diagram: (a) his weight; (b) the force exerted on him by the ground; (c) the force he exerts on the ground; (d) the tension in the calf muscles?

Answers to Odd-Numbered Questions

1 None, since there is no horizontal acceleration; **3** yes; **5** worse; **7** b; **9** (a) no; (b) yes; **11** b; **13** c; **15** If it sags a little.

Problems (listed by section)

4-4   Mass

1 The same force that gives the standard 1 kg mass an acceleration of 1.00 m/s² acts on a body, producing a horizontal acceleration of 1.00×10^{-2} m/s². No other horizontal force acts on the body. Find its mass in kg.

2 The same force that gives the standard 1 kg mass an acceleration of 1.00 m/s² acts first on body A, producing an acceleration of 0.500 m/s², and then on body B, producing an acceleration of 0.333 m/s². Find the acceleration produced when A and B are attached and the same force is applied.

4-5   Newton's Second and Third Laws

(Unless otherwise stated, all systems are assumed to be viewed from an inertial reference frame.)

3 Is the particle shown in Fig. 4-26 accelerated?
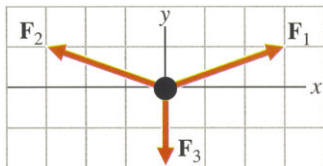
Fig. 4-26

4 Is the particle shown in Fig. 4-27 accelerated?
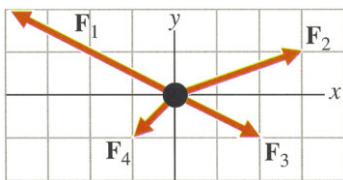
Fig. 4-27

5 The particle shown in Fig. 4-28 is at rest. Find the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.
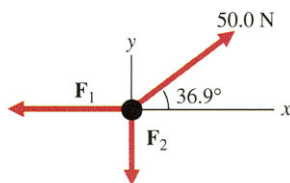
Fig. 4-28

6 The particle shown in Fig. 4-29 is at rest. Find the magnitude and direction of $\mathbf{F}$.
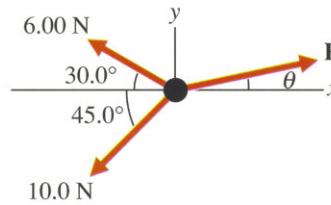
Fig. 4-29

7 A boat is pulled at constant velocity by the two forces shown in Fig. 4-30. Find the horizontal force exerted on the boat by the water.
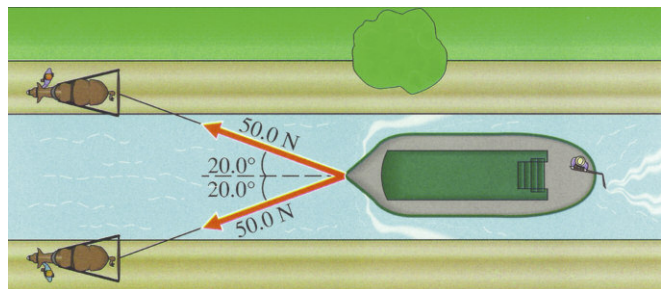
Fig. 4-30

8 A log is dragged along the ground at a constant speed by a force of 425 N at an angle of 45.0° above the horizontal. Find the horizontal component of force exerted by the ground on the log.

9 The canvas tarpaulin shown in Fig. 4-31 is stretched by horizontal forces applied by means of ropes. Find the x and y components of $\mathbf{F}$.
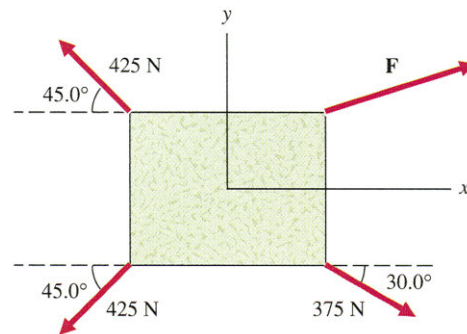
Fig. 4-31

10 A ball is released from rest in an elevator and falls 1.00 m to the floor in 0.400 s. Is the elevator an inertial reference frame?

11 Three children fight over a small stuffed animal of mass 0.200 kg, pulling with the forces indicated in Fig. 4-32. Find the instantaneous acceleration of the toy.
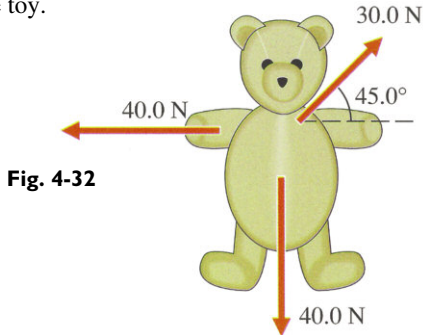

Fig. 4-32

12 Two hockey players strike a puck of mass 0.300 kg with their sticks simultaneously, exerting forces of 1.20×10^3 N, directed west, and 1.00×10^3 N, directed 30.0° east of north. Find the instantaneous acceleration of the puck.

13 A girl scout paddling a canoe pushes the water back with her paddle, exerting a backward force of 155 N on the water. Find the acceleration of the girl and the canoe if their combined mass is 90.0 kg.

14 A golf ball of mass 4.50×10^{-2} kg is struck by a club. Contact lasts 2.00×10^{-4} s, and the ball leaves the tee with a horizontal velocity of 50.0 m/s. Compute the average force the club exerts on the ball by finding its average acceleration.

15 A 3.00 kg mass is acted upon by four forces in the horizontal (xy) plane, as shown in Fig. 4-33. Find the acceleration of the mass.
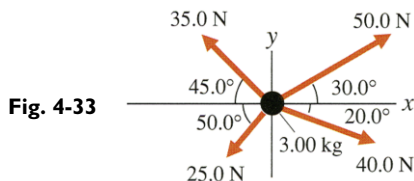
Fig. 4-33

⋆16 A boxer stops a punch with his head. To approximate the force of the blow, treat the opponent's glove, hand, and forearm as a particle of mass 1.50 kg moving with an initial velocity of 20.0 m/s. Estimate the force exerted on the head if (a) the hand moves forward 10.0 cm while delivering the blow and then coming to rest; (b) the head is deliberately moved back during the punch so that the hand moves forward 20.0 cm while decelerating.

17 A boat and its passengers have a combined mass of 5.10×10^2 kg. The boat is coasting into a pier at a speed of 1.00 m/s. How great a force is required to bring the boat to rest in 1.00×10^{-2} s?

⋆18 A 110 kg fullback runs at the line of scrimmage.
   (a) Find the constant force that must be exerted on him to bring him to rest in a distance of 1.0 m in a time interval of 0.25 s.
   (b) How fast was he running initially?

⋆19 A car traveling initially at 50.0 km/h crashes into a brick wall. The front end of the car collapses, and the 70.0 kg driver, held in his seat by a shoulder harness, continues to move forward 1.00 m after the initial contact, decelerating at a constant rate. Find the horizontal force exerted on him by the seat harness.

4-6   Force Laws

Weight

20 (a) Compute your weight in N.
   (b) Compute your mass in kg and in slugs.
   (c) How much weight would you lose in going from the North Pole, where $g = 9.83$ m/s^2, to the equator, where $g = 9.78$ m/s^2, assuming no loss in mass?

21 (a) A 1.00 kg book is held stationary in the hand. Find the forces acting on the book and the reaction forces to each of these.
   (b) The hand now exerts an upward force of 15.0 N on the book. Find the book's acceleration.
   (c) As the book moves upward, the hand is quickly removed from the book. Find the forces on the book and its acceleration.

22 Find the vertical force exerted by the air on an airplane of mass 5.00×10^4 kg in level flight at constant velocity.

23 Just after opening a parachute of negligible mass, a parachutist of mass 90.0 kg experiences an instantaneous upward acceleration of 1.00 m/s^2. Find the force of the air on the parachute.

Tension

⋆24 A small weight hangs from a string attached to the rearview mirror of a car accelerating at the rate of 1.00 m/s^2. What angle does the string make with the vertical?

25 In a tug of war, two teams pull on opposite ends of a rope, attempting to pull the other team across a dividing line. Team A accelerates toward team B at the rate of 0.100 m/s^2. Find all the horizontal forces acting on each team if the weight of each team is 1.00×10^4 N and the tension in the rope is 5.00×10^3 N.

Spring Force

26 The block in Fig. 4-34 rests on a frictionless surface. Find its instantaneous acceleration when the spring on the left is compressed 5.00 cm while the spring on the right is stretched 10.0 cm. Each spring has a force constant of 1.00×10^3 N/m.
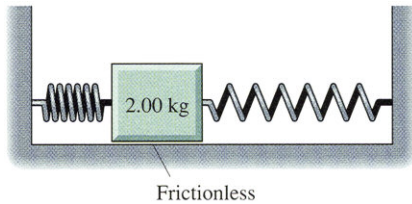

Fig. 4-34

★**27** When a 0.100 kg mass is suspended at rest from a certain spring, the spring stretches 4.00 cm. Find the instantaneous acceleration of the mass when it is raised 6.00 cm, compressing the spring 2.00 cm.

4-8 Applications of Newton's Laws of Motion

28 Find the tension in the ropes shown in Fig. 4-35 at points A, B, C, D, and E. The pulleys have negligible mass.
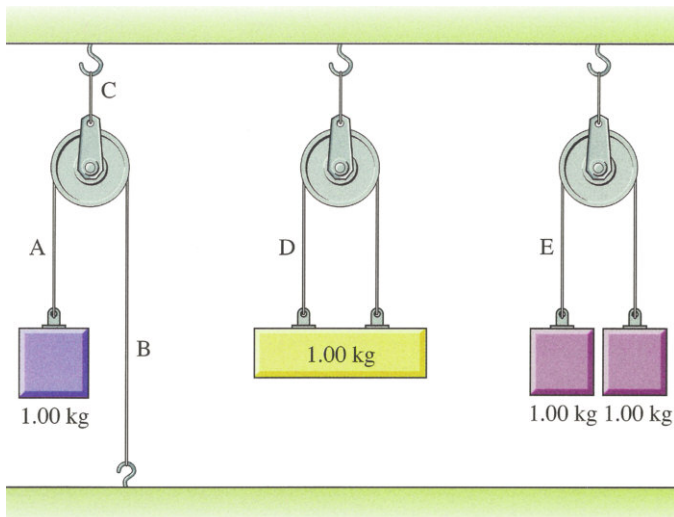
Fig. 4-35

29 Find the tension in each string in Fig. 4-36.
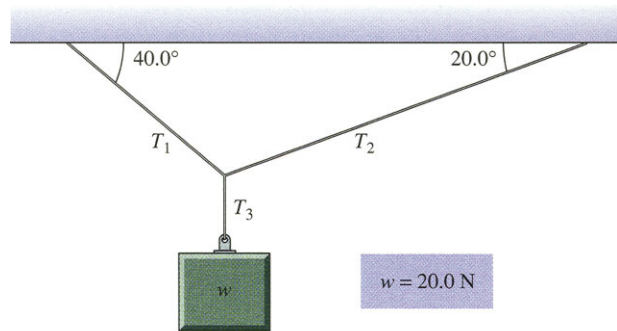
Fig. 4-36

★**30** Find the tension in each string in Fig. 4-37.
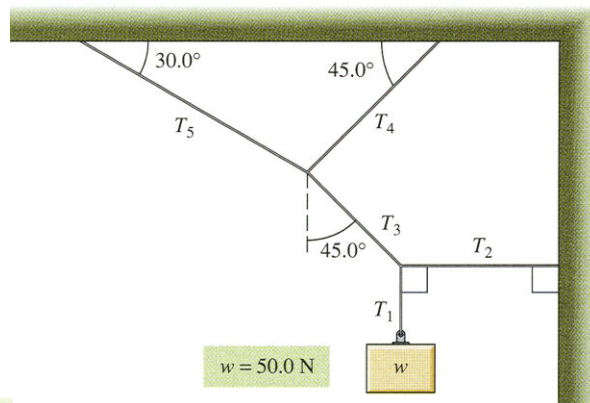
Fig. 4-37

31 A crate weighing 5.00×10^2 N is lifted at a slow, constant speed by ropes attached to the crate at A and B (Fig. 4-38). These two ropes are joined together at point C, and a single vertical rope supports the system. (a) Find the tension T_1 in the vertical rope. (b) Find the tensions T_2 and T_3 in the other ropes.
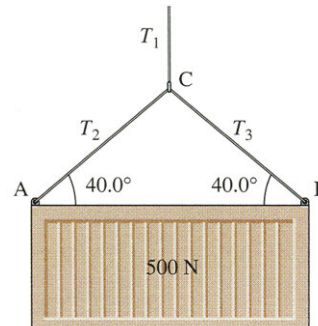
Fig. 4-38

32 A picture of width 40.0 cm, weighing 40.0 N, hangs from a nail by means of flexible wire attached to the sides of the picture frame. The midpoint of the wire passes over the nail, which is 3.00 cm higher than the points where the wire is attached to the frame. Find the tension in the wire.

33 Three blocks are suspended at rest by the system of strings and frictionless pulleys shown in Fig. 4-39. What are the weights w_1 and w_2?
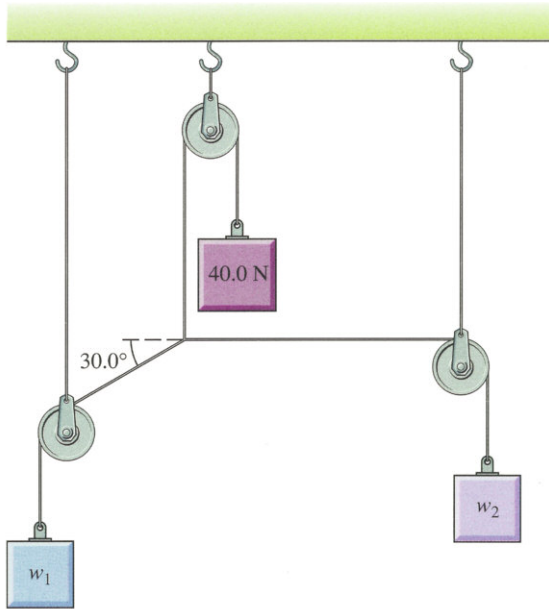

Fig. 4-39

★ **34** Find θ and w in Fig. 4-40, assuming that the arrangement is at rest.
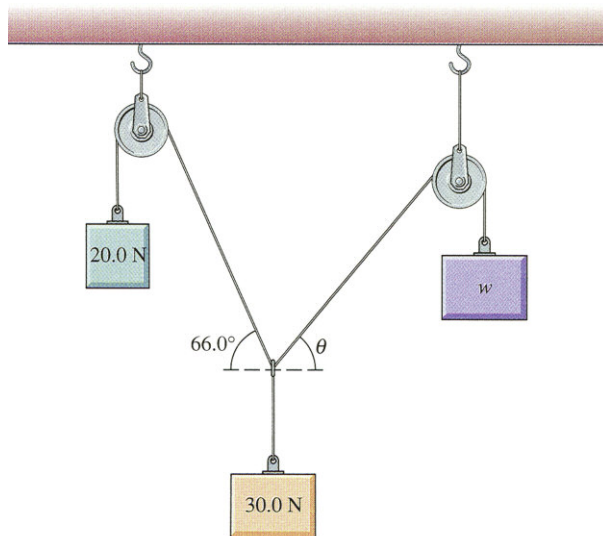
Fig. 4-40

35 A person weighing 710 N lies in a hammock supported on either end by ropes that are at angles of 45° and 30° with the horizontal (Fig. 4-41). Find the tension in the ropes.
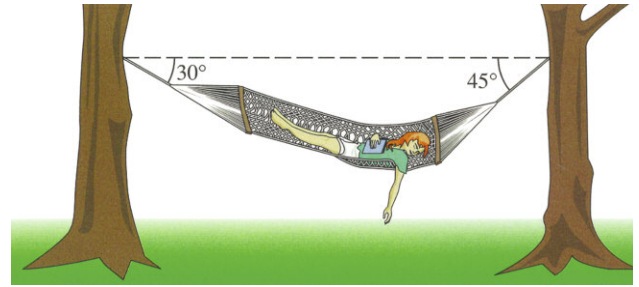
Fig. 4-41

36 Compute the acceleration of each mass in Fig. 4-23a and the tension in the rope. Let $m_1 = 1.00$ kg and $m_2 = 2.00$ kg.

37 Two blocks are connected by a string and are pulled vertically upward by a force of 165 N applied to the upper block, as shown in Fig. 4-42.
  (a) Find the tension T in the string connecting the blocks.
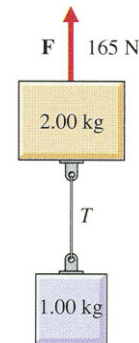 (b) If the blocks start from rest, what is their velocity after having moved a distance of 10.0 cm?

Fig. 4-42

★ **38** Two blocks are initially at rest on frictionless surfaces and are connected by a string that passes over a frictionless pulley (Fig. 4-43). Find the tension in the string.
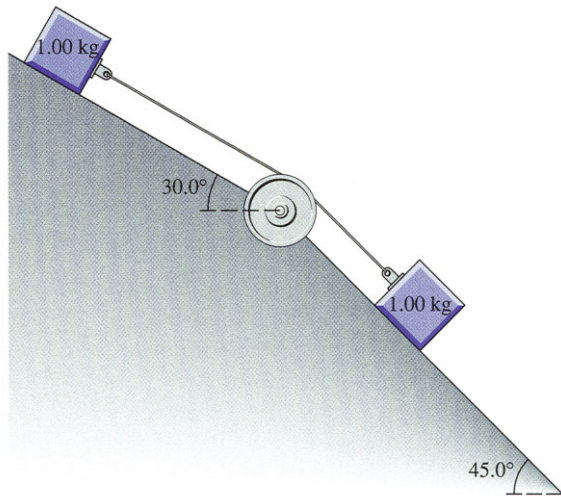

Fig. 4-43

★ **39** Two blocks connected by a string are on a horizontal frictionless surface. The blocks are connected to a hanging weight by means of a string that passes over a pulley (Fig. 4-44).
(a) Find the tension T in the string connecting the two blocks on the horizontal surface.
(b) How much time is required for the hanging weight to fall 10.0 cm if it starts from rest?
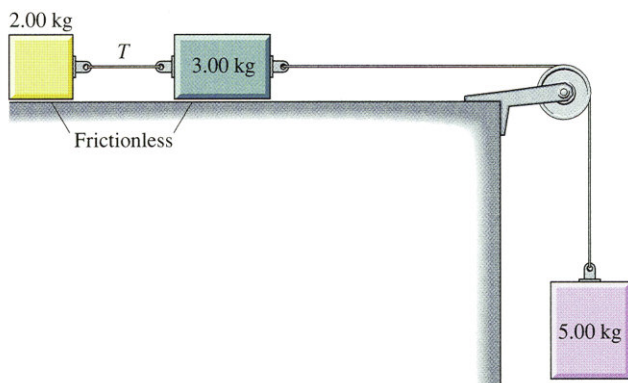
Fig. 4-44

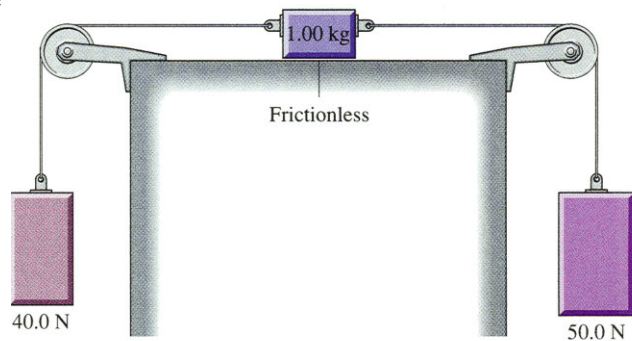★ **40** Find the acceleration of the 1.00 kg block in Fig. 4-45.

Fig. 4-45

★ **41** Two children of equal weight are suspended on opposite ends of a rope hanging over a pulley. Child A begins to slide down the rope, accelerating downward at a rate of 2.00 m/s². Find the direction and magnitude of child B's acceleration, assuming B doesn't slide.

42 A jet airplane has an instantaneous acceleration of 2.00 m/s² at an angle of 20.0° above the horizontal. Compute the horizontal and vertical components of force exerted on a 50.0 kg passenger by the airplane seat.

★★ **43** A boy weighing 4.00×10^2 N jumps from a height of 2.00 m to the ground below. Assume that the force of the ground on his feet is constant.
(a) Compute the force of the ground on his feet if he jumps stiff-legged, the ground compresses 2.00 cm, and the compression of tissue and bones is negligible.
(b) Compute the force his legs exert on his upper body (trunk, arms, and head), which weighs 2.50×10^2 N, under the conditions assumed above.
(c) Now suppose that his knees bend on impact, so that his trunk moves downward 40.0 cm during deceleration. Compute the force his legs exert on his upper body.

★ **44** A car is stuck in a mudhole. In order to move the car, the driver attaches one end of a rope to the car and the other end to a tree 10.0 m away, stretching the rope as much as possible (Fig. 4-46). The driver then applies a horizontal force of 4.00×10^2 N perpendicular to the rope at its midpoint. The rope stretches, with its center point moving 50.0 cm to the side as a result of the applied force. The car begins to move slowly. What is the tension in the rope?
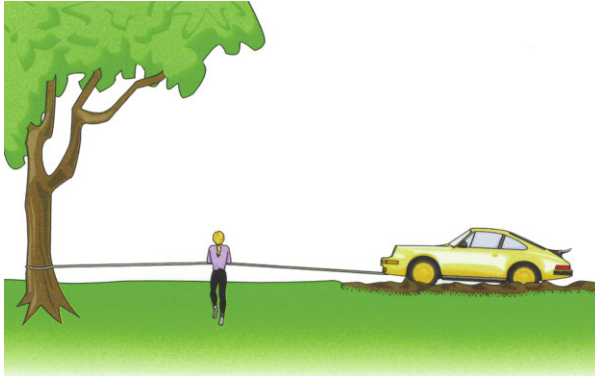

Fig. 4-46

Additional problems

★ **45** A painter on a platform raises herself by pulling on a rope connected to a system of pulleys (Fig. 4-47). If the painter and the platform combined weigh 1050 N, what force must she exert on the rope in order to raise herself slowly?
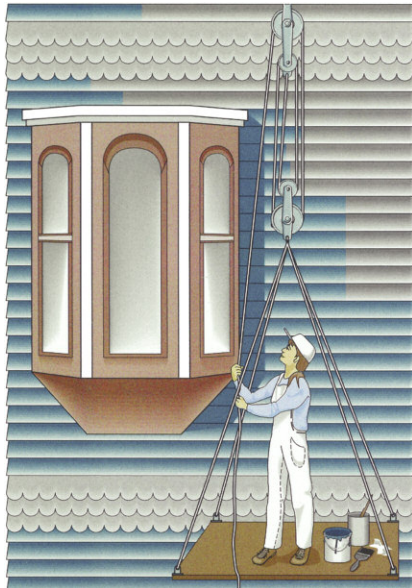

Fig. 4-47

★ **46** Three blocks, each having a mass of 1.00 kg, are connected by rigid rods of negligible mass and are supported by a frictionless surface. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$, of magnitude 5.00 N and 10.0 N respectively, are applied to the ends of the blocks (Fig. 4-48). Find the forces acting on block B.
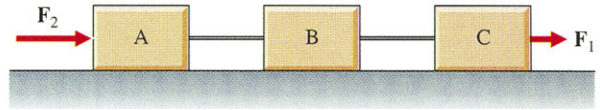

Fig. 4-48

★ **47** A football of mass 0.420 kg is thrown 60.0 m by a quarterback who imparts to it an initial velocity at an angle of 45.0° above the horizontal. If the quarterback moves his hand along an approximately linear path of length 40.0 cm while accelerating the football, what force does his hand exert on the ball, assuming the force to be constant?

48 A wet shirt weighing 5.00 N hangs from the center of a 10.0 m long clothesline, causing it to sag 5.00 cm below the horizontal. Find the tension in the line.

★ **49** A Ping-Pong ball is given an upward initial velocity. The force of air resistance causes the times of ascent and descent to be unequal. Which time is greater?

50 A basketball player stands in front of a basket and, without bending his knees, jumps straight up. The player weighs 1.00×10^3 N. His feet push downward on the floor with a constant force of 2.00×10^3 N for a time interval of 0.100 s, after which they leave the floor. Find (a) his acceleration while his feet are in contact with the floor; (b) his body's velocity as his feet leave the floor; (c) the maximum height he moves upward during the jump.

★★ **51** Each of the two identical springs in Fig. 4-49 has force constant $k = 1.00 \times 10^3$ N/m.
(a) Find the unstretched length of each spring.
(b) Find the instantaneous acceleration of the weight if it is pulled 10.0 cm lower and released.

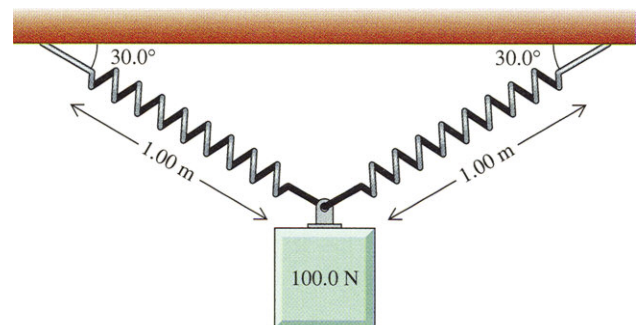**Fig. 4-49**

★★ 52 Consider two particles, of mass m and m', having a common acceleration $\mathbf{a}$, as shown in Fig. 4-50. These particles are subject to equal magnitude internal forces $\mathbf{F}_i$ and $\mathbf{F}'_i$, and to external forces $\mathbf{F}_e$ and $\mathbf{F}'_e$. Show that it follows from Newton's second law applied to each particle separately that we may apply the second law to the system of two particles if we use the net external force, the combined mass $m + m'$, and the common acceleration $\mathbf{a}$.
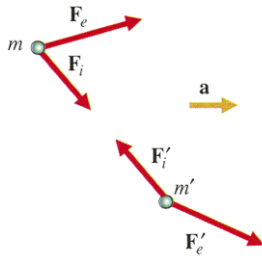
Fig. 4-50

★★ 53 Find the angles θ_1 and θ_2 in Fig. 4-51 if all the weights are at rest.
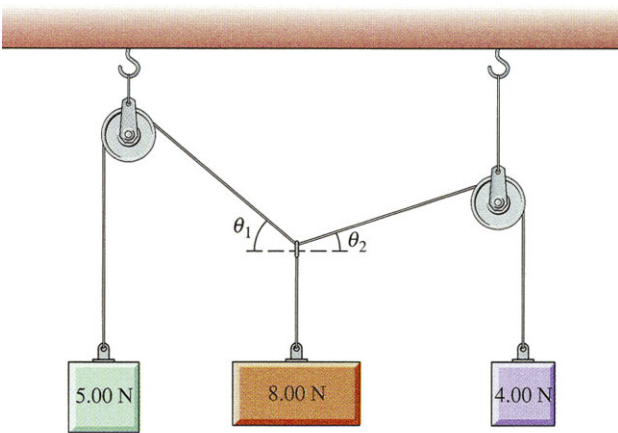
Fig. 4-51

★ 54 The force of air resistance $\mathbf{R}$ on a freely falling body is in a direction opposing the velocity and has magnitude approximately given by $R = CAv^2$, where A is the cross-sectional area of the body in the plane perpendicular to the motion, v is the speed, and C is a constant depending on body shape and air density. Show that if the y-axis is taken to be positive in the downward direction, a falling body experiences an acceleration $a_y = g(1 - CAv_y^2/w)$. Show that as a_y approaches zero, v_y approaches terminal velocity $v_T = \sqrt{w/CA}$.

★★ 55 A runner moving through the air experiences a force $\mathbf{R}$ because of the air (Fig. 4-52). This force, which is a function of the runner's velocity relative to the air, is approximately proportional to the square of the relative speed v_r. The magnitude of force $\mathbf{R}$ may be expressed as $R = CAv_r^2$, where A is the cross-sectional area of the body in the plane perpendicular to the motion. Suppose a runner first moves a distance D along the ground at constant velocity in the direction of a steady wind and then moves the same distance in the opposite direction at the same speed with respect to the ground. For both parts of the motion, express R in terms of the runner's speed v (relative to the ground) and the speed of the wind v_w. By how much does the average of these two values exceed the average magnitude of $\mathbf{R}$ in the absence of wind? This problem illustrates how wind generally produces a higher average value of air resistance on a runner, even though the runner runs with the wind the same distance he or she runs against the wind.

Fig. 4-52

★ 56 In Fig. 4-53 the mass of block A is 10.0 kg and that of block B is 15.0 kg. The pulley is massless and frictionless.

(a) What is the largest vertical force $\mathbf{F}$ that can be applied to the axle of the pulley if B is to remain on the floor?

(b) What will be the acceleration of A when this maximum force is applied?

Fig. 4-53