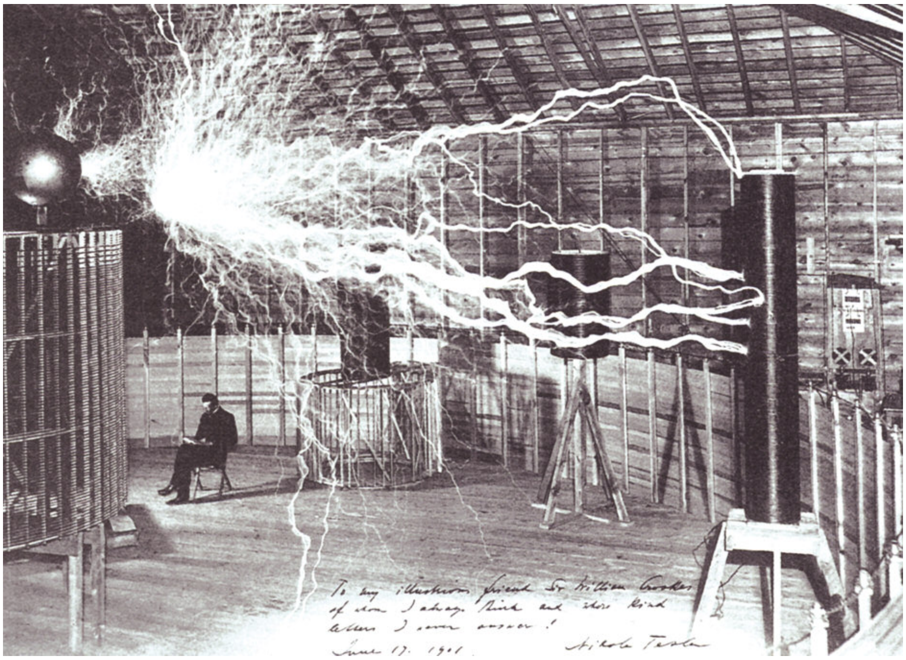
CHAPTER 17

The Electric Field



This electrical discharge was produced at the turn of the 20th century by inventor Nikola Tesla, shown in the photo.

You can observe "static electricity" in various ways. Walk across a wool rug with rubber-soled shoes on a dry day and your body becomes "charged," and then, when you touch a metal doorknob, there is a sudden, painful spark. Remove clothes from a dryer, and they cling together because electric charges have been transferred between the clothes. Brush your hair on a very dry day, and both hair and brush become charged. The brush then attracts dust or small bits of paper and your hair stands on end. All these phenomena result from forces acting between electric charges at rest.

This chapter and the next deal with **electrostatics,** the study of electric charges at rest. This will serve as a foundation for understanding electric current (charges in motion), with its many applications to modern technology. After studying current, we shall study magnetism and show its connection with electricity. The culmination of all this will be our study of the laws of electromagnetism, in which electric and magnetic phenomena are shown to be intimately connected.

17-1 Electric Charge

Historical Origins

One of the earliest observations of electric attraction was that amber that has been rubbed with a piece of cloth will attract light objects, such as feathers or straw. This was known to the Greeks as early as 600 B.C. In 1600 A.D. William Gilbert, personal physician to Queen Elizabeth I, published the first systematic study of electricity and magnetism. Gilbert showed that, contrary to popular opinion, amber was not unique in its attractive property; many other substances could produce similar attractive effects after frictional contact. This attractive force was obviously a very general property of nature, and so he gave it a special name—"electric force." (*Elektron* is the Greek word for 'amber.')

No further significant progress was made in the science of electricity until 1734, when Charles du Fay attempted to account for his observation that the electric force could be repulsive as well as attractive. Du Fay believed that there must be two kinds of "electrical fluid" that could flow into a body and "electrify" it. Two bodies charged with the same kind of fluid would repel each other, whereas two bodies charged with different kinds of fluid would attract each other.

In 1750 Benjamin Franklin, on the basis of extensive experiments, sought to replace the two-fluid theory by a one-fluid theory. He believed that an excess of this single fluid was responsible for one type of electrical charging and a deficiency of it was responsible for the other type of charging. The body containing the excess fluid he described as "positively charged," and the body with the deficiency he called "negatively charged." For example, Franklin believed that when someone rubs a piece of glass with the bare hand both glass and skin become charged because some of the electric fluid in the skin is transferred to the glass. This movement of the fluid gives the skin a deficiency of the fluid and the glass an excess. Thus he assigned a negative charge to the skin and a positive charge to the glass. Franklin proposed that when two oppositely charged bodies are brought together the body with the excess fluid gives it up to the body with the deficiency; both bodies become uncharged and no longer produce electrical force.

Today we know that matter normally consists of equal quantities of positive and negative electric charges—protons and electrons—and therefore has no net charge. The protons are tightly bound in the nuclei of atoms, whereas the electrons, which are far less massive than the protons, are more weakly bound to atoms and therefore are more easily removed. The masses of the proton and electron are

$$m_{\mathrm{p}} = 1.67 \times 10^{-27}\ \mathrm{kg}$$

$$m_{\mathrm{e}} = 9.11 \times 10^{-31}\ \mathrm{kg}$$

When a body does not contain an equal number of protons and electrons, it has a net positive or negative charge. Transfer of electrons to or from a body gives that body a net charge.* If we think of electrons as Franklin's electrical fluid, we see that his ideas concerning electrical charging were essentially correct. Franklin's analysis of the charging of skin and glass was incorrect in one aspect, however. Frictional contact between skin and glass results in the flow of the "fluid" (electrons) from glass to skin, rather than from skin to glass as supposed by Franklin; hence the skin, to which Franklin assigned a negative charge, has an excess of electrons. It follows that the electrons must have negative charge. This is just a convention, arising from Franklin's arbitrary assignment of negative charge to the skin.

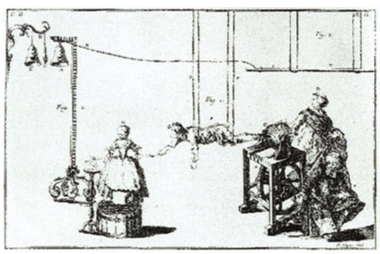


Fig. 17-1 Many of the early experiments with static electricity involved human subjects who were used to detect the presence of electric charge by the electric shock it produced.

*Sometimes a net charge is the result not of electron transfer but of the transfer of either positively charged "ions" (atoms missing one or more electrons) or negatively charged ions (atoms with one or more extra electrons).

Demonstration of Electrostatic Forces

The following simple experiment demonstrates several important properties of electric charge. First charge a piece of aluminum foil by rubbing it with plastic wrap. Then let the aluminum touch two Ping-Pong balls, each suspended from a string. When the balls are brought close together, they experience a mutual repulsive force (Fig. 17-2a). Next charge a piece of rubber by rubbing it with wool, and then let the rubber touch two other Ping-Pong balls. Like the first two, these balls experience a mutual repulsive force (Fig. 17-2b).

If one of the balls charged by rubber is now brought close to one of the balls charged by aluminum, a mutual attractive force is observed (Fig. 17-2c), but if these two balls are allowed to touch each other, the electrical attraction quickly disappears (Fig. 17-2d). (We assume here that the balls are equally charged.)

This behavior can be described in terms of excess and deficiency of electrons. Electrons are taken away from the aluminum when it is rubbed, leaving it with a deficiency of electrons. There are then more protons than electrons and therefore a net positive charge. When the aluminum touches two balls, they become positively charged as some of their electrons flow to the electron-deficient foil. The two positively charged balls (A) repel each other. Electrons from the wool are added to the rubber when it is rubbed, leaving the rubber and the balls it touches negatively charged. The negatively charged balls (R) repel each other. When a positively charged ball (A) is close to a negatively charged ball (R), they attract. When the two are brought into contact, electrons flow from R to A, leaving each electrically neutral, and they no longer experience any force.

This experiment demonstrates an important general property of electric charges: **like charges repel,** and **unlike charges attract.**

An electroscope is a device that uses the repulsion of like charges to show the presence of charge. It consists of a metallic rod with two small pieces of gold leaf at the bottom. When charged, the leaves repel each other (Fig. 17-3).

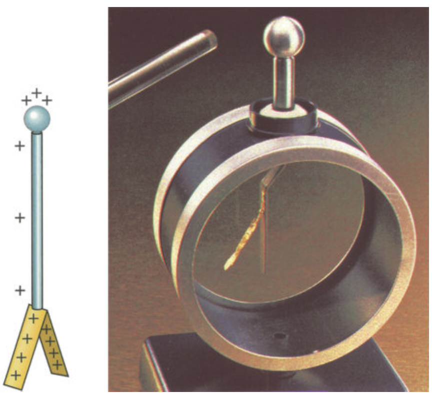


Fig. 17-3 When charged, the leaves of an electroscope stand out.

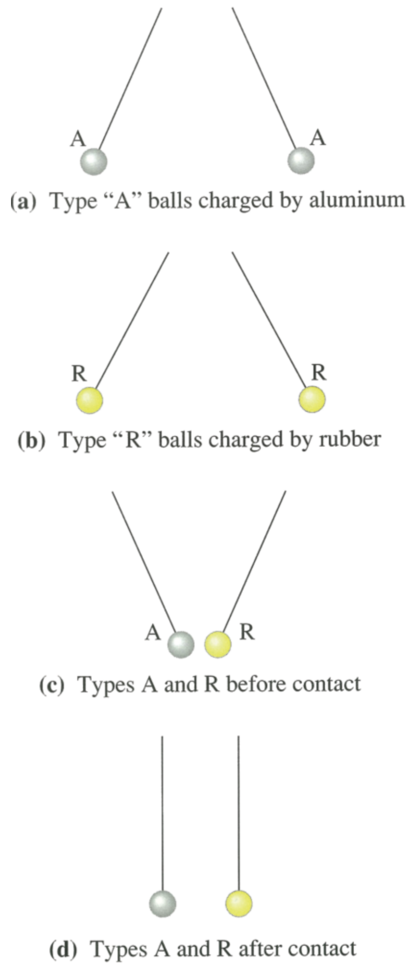


Fig. 17-2 Forces between charged Ping-Pong balls.

Insulators and Conductors

When some materials are charged by contact with a charged object, the excess or deficiency of electrons very quickly leaves the point of contact and distributes itself over the entire surface of the newly charged material. Such materials are called **conductors.** They readily conduct electric charge from one point to another. Metals are good conductors, as are the human body and the earth. For other materials, called **insulators** or **dielectrics,** charge placed on one part of the surface remains localized. Most nonmetals are insulators. For example, wood, rubber, glass, and plastic are all insulators.

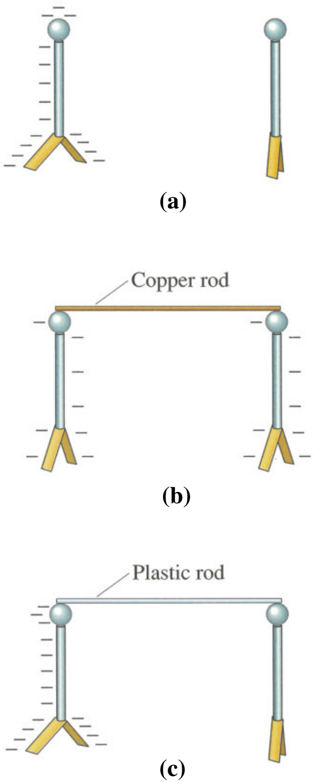


Fig. 17-4 A copper rod transfers charge from one body to another **(b)**, but a plastic rod does not **(c)**.

Metals are effective in transporting electrons from one body to another. Suppose you have a negatively charged body and want to transfer some of its charge to a second, initially uncharged body (Fig. 17-4a). If you connect a copper rod between the two, some of the excess electrons in the first body will pass through the rod and onto the second body (Fig. 17-4b). If, on the other hand, you connect the two bodies by an insulator—a plastic rod, for example—there will be no transfer of charge, and the second body will remain uncharged (Fig. 17-4c).

It is the microscopic structure of insulators and conductors that is responsible for their very different conducting properties. All the electrons in an insulator are bound to individual atoms, but in a conductor each atom gives up some small number of electrons so that these electrons are free to move through the conductor. The remaining electrons and the positively charged nucleus of each atom are bound together to form a positively charged ion. The positive ions are bound together in the conductor in a fixed structure (Fig. 17-5).

Although a conductor may have no net charge, it still contains an enormous reservoir of free electrons that can quickly respond to an electric force imposed by a nearby charged body. These free electrons then move through the conductor.

Conservation of Charge

Although charge moves from place to place, net charge is never created or destroyed at any point in space. This principle is known as "conservation of charge" and is a general law of nature. If electrons and protons were indestructible particles, charge conservation would be obvious. Experiments indicate, however, that these particles are not indestructible. They, along with many other particles, may be created and destroyed. However, in all such experiments there is never creation or destruction of *net* charge. For example, an electron may interact with a "positron" with the result that the two annihilate each other, leaving no charge behind. (The positron is a particle that has exactly the same mass as the electron but carries a positive charge.) Since the electron and positron have opposite charges of equal magnitude, the net charge of the system is zero both before and after the annihilation, and so the principle of charge conservation is satisfied.

Units of Charge

An obvious and natural choice for the basic unit of charge is the magnitude of the electron's charge, which we denote by e. Thus we express the electron's charge as $-e$ and the proton's charge as $+e$:

$$\text{electron charge} = -e$$

$$\text{proton charge} = +e$$

A charged macroscopic body usually has an excess or deficiency of an enormously large number of electrons, and so it is convenient to define a larger unit of charge. The coulomb (abbreviation C) is such a unit. It is an experimentally determined unit* that may be expressed as

$$1\ \text{C} = (6.24 \times 10^{18})\text{e} \tag{17-1}$$

Alternatively, e may be expressed as $1\ \text{C}/6.24 \times 10^{18}$, or

$$\text{e} = 1.60 \times 10^{-19}\ \text{C} \tag{17-2}$$

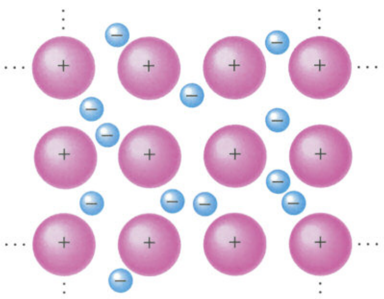


Fig. 17-5 The charge distribution in an uncharged conductor. The positively charged ions are bound to fixed positions, but some electrons are free to move throughout the conductor.

*Eq. 17-1 is not the definition of the coulomb. Instead, the coulomb is defined in terms of the ampere, the SI unit of electric current, which we shall define in Chapter 20.

The symbol q is used to denote charge. The charge on a body can be expressed as an integral multiple of e:

$$q = \pm ne \tag{17-3}$$

where n is the number of electrons that have been either taken from or added to the body.

EXAMPLE 1 Finding the Number of Excess Electrons on a Charged Body

A body has a net charge of -1.00×10^{-9} C. How many excess electrons are contained in the body?

SOLUTION Solving Eq. 17-3 for n, we have

$$n = \frac{-q}{\mathrm{e}} = \frac{-(-1.00 \times 10^{-9}\ \mathrm{C})}{1.60 \times 10^{-19}\ \mathrm{C}}$$

$$= 6.25 \times 10^{9}$$

17-2 Coulomb's Law

Two Point Charges

In 1785 Charles Coulomb established the fundamental force law for two static point charges*:

1. Each of the two charges experiences a force that is directed along the line between the two charges; the force is repulsive for charges of like sign and attractive for charges of opposite sign (Fig. 17-6).
2. The magnitude of the force is proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance r between them:

$$F = k\frac{|q||q'|}{r^2} \tag{17-4}$$

where k is a force constant that experiment shows to have the value

$$k = 8.99 \times 10^9\ \text{N-m}^2/\text{C}^2$$

In problem solving, we shall sometimes round this off to $k = 9.0 \times 10^9$ N-m²/C².

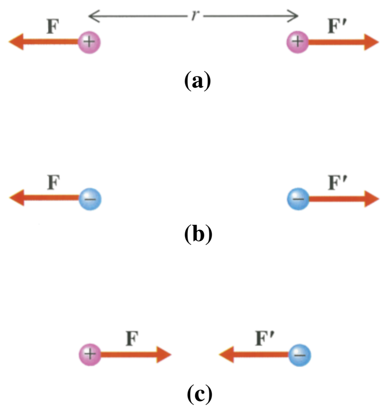


Fig. 17-6 Forces between charges.

*Coulomb used a torsion balance to perform his experiments. This device was imitated by Cavendish years later to study gravitational force. The Cavendish torsion balance was described in Chapter 6.

EXAMPLE 2 Force Exerted by One Charge on Another

Two point charges, $q = +1.00$ C and $q' = -1.00$ C, are located 2.00 m apart (Fig. 17-7a). What are the magnitude and direction of the force that q exerts on q'?

SOLUTION Since the charges are of opposite sign, the force between them is attractive, and so the force **F** that q exerts on q' is directed toward q, as shown in Fig. 17-7b. (Of course there is also an oppositely directed reaction force acting on q, but we are not asked to compute it, and so we have not shown it in the figure.)

The magnitude of **F** is found from Eq. 17-4:

$$F = k\frac{|q||q'|}{r^2} = (8.99 \times 10^9 \text{ N-m}^2/\text{C}^2)\frac{(1.00 \text{ C})(1.00 \text{ C})}{(2.00 \text{ m})^2}$$

$$= 2.25 \times 10^9 \text{ N}$$

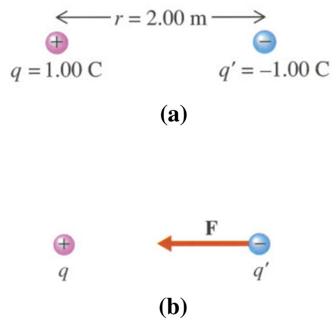


Fig. 17-7

This is an enormously large force, and thus we would normally never observe such large concentrations of charge. In ordinary static charging of bodies through friction, the charge might typically be something like 10^{-10} C.

Coulomb's Law and Universal Gravitation

Coulomb's law is the second fundamental force law we have encountered in our study of physics. It is similar in form to the fundamental force law introduced in Chapter 6, Newton's law of universal gravitation. Recall that the gravitational force between two particles is proportional to the product of their masses and inversely proportional to the square of the distance between them (Eq. 6-1):

$$F = G\frac{mm'}{r^2} \tag{17-5}$$

where $$G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$$

Thus the Coulomb force depends on charge in the same way that the gravitational force depends on mass, and both forces have the same $1/r^2$ dependence.

There are, of course, important differences between Coulomb's law and the gravitational force law. Although there is only one kind of mass, there are two kinds of charge, positive and negative, and although the gravitational force is always attractive, the electrostatic force may be attractive or repulsive, depending on the signs of the charges.

Another obvious difference between the two force laws is that the electrical force constant k is much larger than the gravitational constant G. Thus, as we saw in Example 2, two bodies 2.00 m apart, each carrying a charge of magnitude 1.00 C, experience a force of 2.25×10^9 N, or about half a billion pounds. On the other hand, two 1.00 kg bodies 2.00 m apart experience a mutual gravitational force of only 1.67×10^{-11} N. The large value of the electric force constant means that the interaction of even relatively small charges can produce significant forces.

Superposition Principle

When several point charges are present in a region of space, **each charge experiences forces exerted by the other charges. The resultant force acting on each charge is the *vector* sum of all the Coulomb forces acting on that charge**. This principle is called the **superposition principle** and is illustrated for three positive charges in Fig. 17-8.

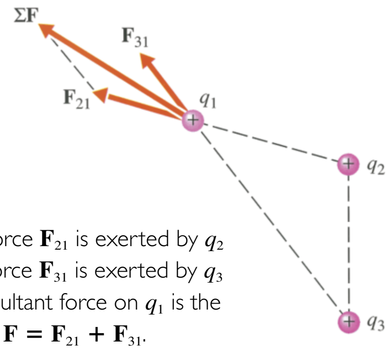


Fig. 17-8 Force $\mathbf{F}_{21}$ is exerted by q_2 on q_1, while force $\mathbf{F}_{31}$ is exerted by q_3 on q_1. The resultant force on q_1 is the vector sum $\Sigma\,\mathbf{F} = \mathbf{F}_{21} + \mathbf{F}_{31}$.

EXAMPLE 3 The Resultant of Two Electrostatic Forces on a Charge

Find the resultant force on q_3 in Fig. 17-9a.

SOLUTION The first step is to indicate in a figure the forces $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ exerted on q_3 by q_1 and q_2 respectively (Fig. 17-9b). Notice that $\mathbf{F}_{13}$ is drawn with the tail of the arrow on q_3, since that is the charge this force acts on, and is directed away from q_1, since both q_1 and q_3 are positive and charges of like sign repel. Force $\mathbf{F}_{23}$ is directed toward q_2, since q_2 and q_3 have opposite signs and q_2 therefore exerts an attractive force on q_3. We have also indicated in the figure the distances from q_3 to the other two charges.

The next step is to calculate the magnitudes of the forces. Applying Coulomb's law (Eq. 17-4), we find

$$F_{13} = k\frac{|q_1||q_3|}{r^2}$$

$$= (9.0 \times 10^9\ \text{N-m}^2/\text{C}^2)\frac{(1.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{(3.0 \times 10^{-2}\ \text{m})^2}$$

$$= 50\ \text{N}$$

$$F_{23} = k\frac{|q_2||q_3|}{r^2}$$

$$= (9.0 \times 10^9\ \text{N-m}^2/\text{C}^2)\frac{(4.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{(3.0\sqrt{2} \times 10^{-2}\ \text{m})^2}$$

$$= 100\ \text{N}$$

The final step is to find the resultant force on q_3 by taking the sum of vectors $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$, using the usual method of vector addition by components. (It may be helpful to review Section 1-5.)

$$\Sigma\,F_x = +F_{13} - F_{23}\cos 45° = 50\ \text{N} - (100\ \text{N})(\cos 45°)$$

$$= -21\ \text{N}$$

$$\Sigma\,F_y = +F_{23}\sin 45° = (100\ \text{N})(\sin 45°)$$

$$= 71\ \text{N}$$

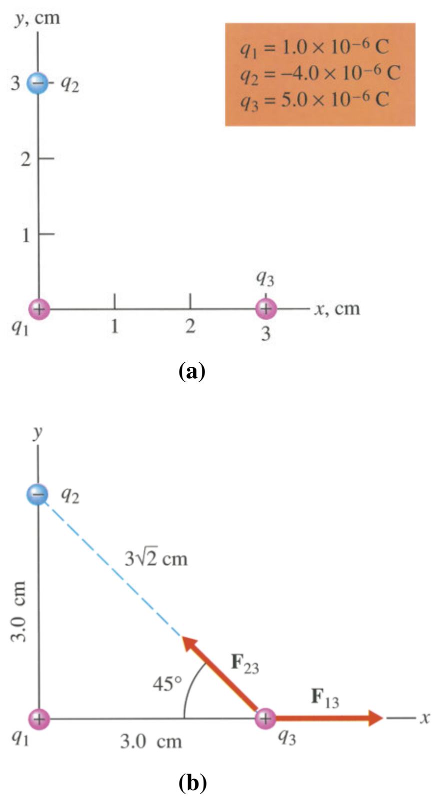


Fig. 17-9

The resultant force has a magnitude given by

$$|\Sigma\,\mathbf{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-21\ \text{N})^2 + (71\ \text{N})^2}$$

$$= 74\ \text{N}$$

This force is directed at an angle θ above the negative x-axis, where

$$\theta = \arctan\frac{71\ \text{N}}{21\ \text{N}} = 74°$$

17-3 The Electric Field

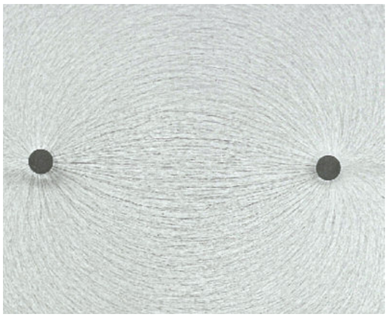


Fig. 17-10 One can show the electric field by suspending bits of thread in oil around electric charges. The threads align with the electric field.

The Concept of a Field

There is another way to view the interaction between charges. Rather than considering directly the forces acting between charges, we can instead introduce the concept of an **electric field.** A charge creates an electric field in the region of space around the charge (Fig. 17-10). The electric field may then act on other charges in that region. Gravitational force may be regarded in a similar way. The earth sets up a gravitational field in space. When a mass is placed near the earth, the earth's gravitational field acts on the mass, and the mass experiences a force.

If you have seen the "Star Trek" television series, you may recall an often repeated scene that can help you understand the concept of an electric field. When the starship Enterprise is in danger of attack, Captain Kirk activates an invisible "force field" around the ship. If any incoming missiles enter that force field, they experience a force, causing them to explode before they reach the ship. Like the Enterprise's force field, an electric field is an invisible force field—one that exerts force on electric charges entering the field.

The electric field concept is an alternative to Coulomb's law for viewing the interaction between charges. For example, in the case of two interacting charges, you can either think of the Coulomb force between the two charges, or think of one charge as the source of an electric field that exerts a force on the second charge. As long as all charges are at rest, the two approaches are equivalent.

Introducing the electric field may seem like an unnecessary complication, since we replace something fairly simple and direct, the Coulomb force between charges, by a less direct approach. However, the field approach turns out to be absolutely essential in later chapters, when we need to describe moving charges. When charges move, their fields change, but the effect is not immediately communicated to other charges. Instead, changes in the field propagate through space at the speed of light. The field affecting a given charge at a certain instant is created by another charge that produced the field at an earlier time. For example, the motion of electrons in a television or radio transmitting antenna creates an electric field. The effect of this electric field can be experienced for many miles around the transmitting antenna. When a television or radio in the vicinity is tuned to that particular station, the electrons in the receiving antenna experience a force that, with the help of the receiver's circuits, results in the images and/or sounds produced (Fig. 17-11). The electric field to which the receiving antenna's electrons are responding at any instant was produced earlier by the motion of the transmitting antenna's electrons.

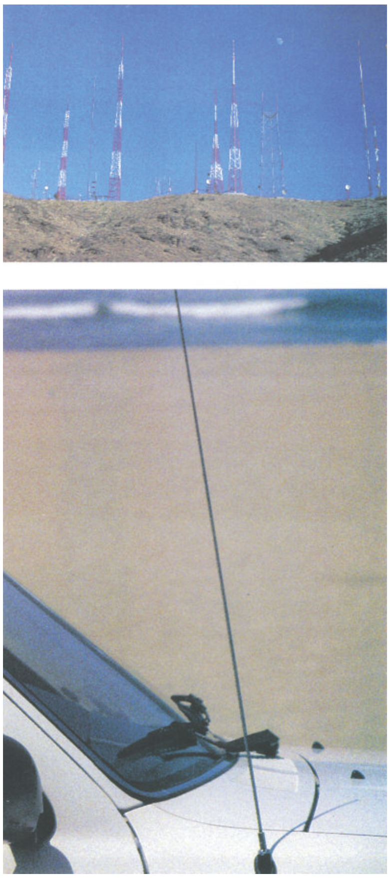


Fig. 17-11 Electrons in a radio receiving antenna respond to the electric field produced by electrons in a distant transmitting antenna.

Definition of the Electric Field

We define the electric field E at a point in space to be the force per unit charge that a test charge would experience if placed at that point. Denoting the test charge by q', we have

$$\mathbf{E} = \frac{\mathbf{F}}{q'} \tag{17-6}$$

From this definition it follows that the electric field is a vector quantity and that the SI unit for the electric field is the force unit divided by the charge unit: N/C.

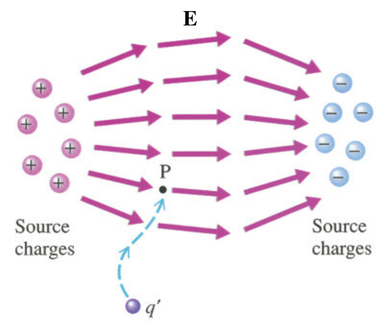
It is important to understand that although the test charge q' enters into the formal definition of the electric field the presence of an electric field at a given point in space does *not* depend on the presence of a test charge at that point. The electric field is the force per unit charge a test charge experiences *if* placed in the field. The electric field is present whether or not there is a test charge to experience its effect.

The source of the electric field may be a single charge or any number of charges (Fig. 17-10). In this section we develop methods for calculating electric fields.

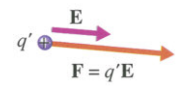
If the electric field at a given point in space is known, we can use the equation defining **E** to find the force **F** on *any* charge q' placed at that point.* Solving Eq. 17-6 for **F**, we obtain

$$\mathbf{F} = q'\mathbf{E} \tag{17-7}$$

Fig. 17-12 illustrates how an electric field is produced by source charges and how this field produces a force on other charges placed in it.



(a) Place charge q' at point P and it experiences a force $\mathbf{F} = q'\mathbf{E}$



(b) Force on q' when q' is placed at P, assuming q' is postive

Fig. 17-12 Source charges produce an electric field **E** that exerts a force **F** on a charge q' placed in the field.

Single Source Charge

We shall begin by showing how to determine the strength and direction of the electric field produced by a single point charge q. We imagine placing a positive test charge q' at any point P near the source charge q (Fig. 17-13a). The point P where we place the test charge is called the **field point,** defined as that point at which we wish to determine the electric field.

If the source charge q is positive, q' is repelled by it. The electric field is defined as the force per unit charge on q' ($\mathbf{E} = \mathbf{F}/q'$), and so the electric field is in the same direction as the force on q'; that is, **the electric field produced by a positive source charge is directed away from the source charge** (Fig. 17-13b).

If the source charge is negative, q' is attracted to it. It follows that **the electric field produced by a negative source charge is directed toward the source charge** (Fig. 17-13c).

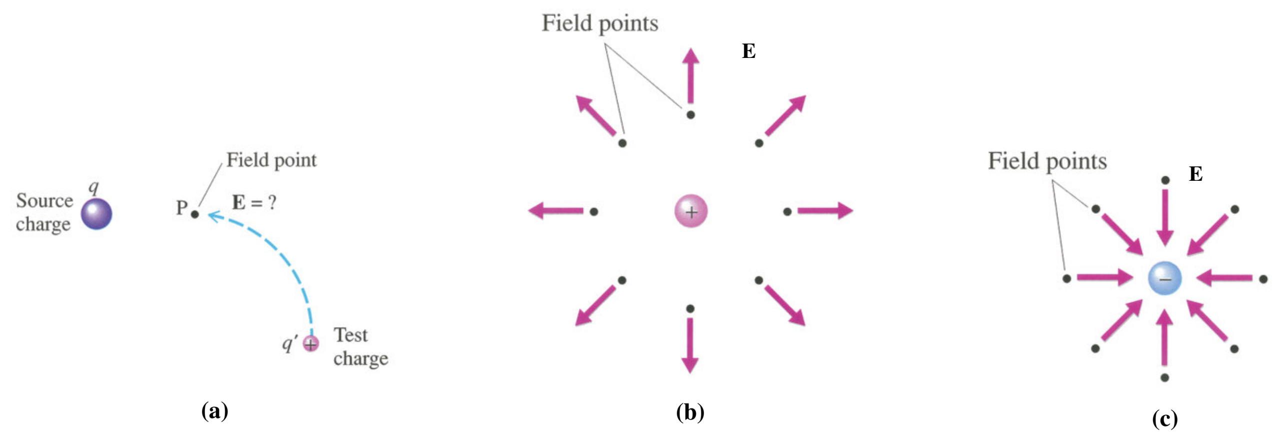


Fig. 17-13 The electric field produced by a single point charge at various field points. **(a)** A test charge at P responds to the field at that point. **(b)** The electric field of a positive charge is directed away from the charge. **(c)** The electric field of a negative charge is toward the charge.

*We assume that the presence of q' does not affect the distribution of charge that is the source of the electric field; either q' is so small that it exerts a negligibly small force on the source charges, or the source charges are somehow held in place.

The magnitude of the force q exerts on q' is given by Coulomb's law (Eq. 17-4):

$$F = k\frac{|q||q'|}{r^2}$$

The electric field is defined as the force per unit charge exerted on a test charge q', and so we find the magnitude of the electric field produced by q by dividing both sides of the preceding equation by $|q'|$:

$$E = k\frac{|q|}{r^2} \tag{17-8}$$

This equation gives the magnitude of the electric field produced by a point charge q at any distance r from the charge.

EXAMPLE 4 The Electric Field of a Negative Charge at Various Field Points

Find the magnitude and direction of the electric field produced by a source charge $q = -1.00 \times 10^{-9}$ C (a) at field point a, located 1.00 m to the right of q, and (b) at field point b, located 2.00 m to the left of q. (c) Find the force on either a $+1.00 \times 10^{-12}$ C charge or a -1.00×10^{-11} C charge placed at field point a.

SOLUTION **(a)** Applying Eq. 17-8 we find

$$E = \frac{k|q|}{r^2} = (8.99 \times 10^9 \text{ N-m}^2/\text{C}^2)\frac{1.00 \times 10^{-9}\text{ C}}{(1.00\text{ m})^2}$$

$$= 8.99 \text{ N/C}$$

Since the source charge is negative, the field is directed toward it, that is, to the left (Fig. 17-14).

(b) Again applying Eq. 17-8, we find

$$E = \frac{k|q|}{r^2} = (8.99 \times 10^9 \text{ N-m}^2/\text{C}^2)\frac{1.00 \times 10^{-9}\text{ C}}{(2.00\text{ m})^2}$$

$$= 2.25 \text{ N/C}$$

At this field point, the field is directed to the right, which is again toward the negative source charge.

The electric field has the same magnitude at all points that are the same distance r from the source charge. The field is thus spherically symmetric. Fig. 17-14 shows the electric field at a and b, as well as at other selected field points at distances of 1 m and 2 m from q. A positive source charge would produce a similar electric field pattern, except that the direction of **E** would be reversed.

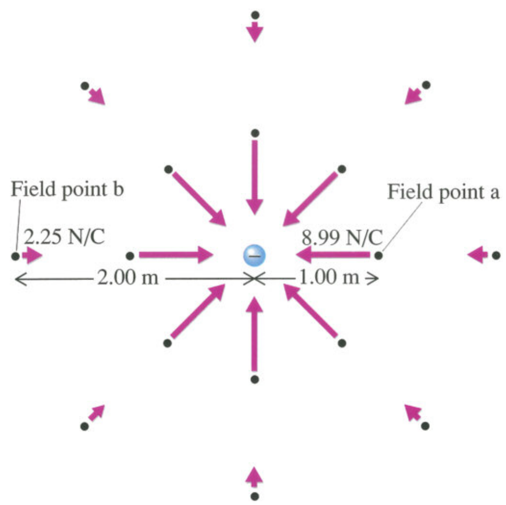


Fig. 17-14

(c) If we place a charge q' in the electric field, according to Eq. 17-7, the charge experiences a force $\mathbf{F} = q'\mathbf{E}$. Thus a charge of $+1.00 \times 10^{-12}$ C placed at a, where $\mathbf{E} = 8.99$ N/C to the left, would experience a force to the left of magnitude 8.99×10^{-12} N. A charge of -1.00×10^{-11} C placed at a would experience a force to the right (in the $-\mathbf{E}$ direction) of magnitude 8.99×10^{-11} N.

EXAMPLE 5 The Gravitational Field of the Earth

The gravitational field is defined as the gravitational force per unit mass that would be experienced by a test mass placed at the point where the field is to be evaluated. Find the magnitude and direction of the gravitational field of the earth at field points that are at distances R and $2R$ from the center of the earth, where R is the earth's radius. Use 6.0×10^{24} kg as the mass of the earth and 6.4×10^6 m as its radius.

SOLUTION The earth is a spherically symmetric distribution of mass M that exerts an attractive force **F** on any other mass m. As seen in Chapter 6, this force is directed toward the center of the earth and has magnitude

$$F = G\frac{mM}{r^2}$$

where G is the gravitational constant (6.67×10^{-11} N-m²/kg²) and r is the distance from m to the center of the earth. Applying the definition of the gravitational field, which we denote by **g**, we find

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

The gravitational field equals gravitational acceleration. The units are either N/kg or, equivalently, m/s². At $r = R$, we find

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2)(6.0 \times 10^{24}\ \text{kg})}{(6.4 \times 10^6\ \text{m})^2}$$

$$= 9.8\ \text{N/kg}$$

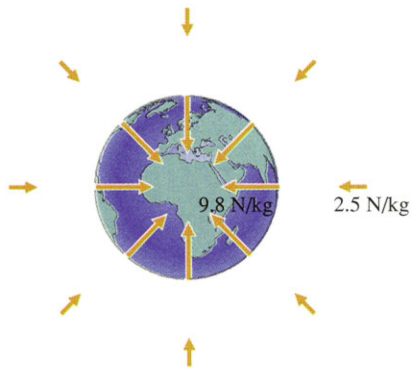


Fig. 17-15

And at $r = 2R$, we find

$$g = \frac{GM}{(2R)^2} = \frac{1}{4}\left(\frac{GM}{R^2}\right) = \tfrac{1}{4}(9.8\ \text{N/kg})$$

$$= 2.5\ \text{N/kg}$$

Fig. 17-15 shows the earth's gravitational field evaluated at selected field points at distances R and $2R$ from the center. The field pattern is the same as for a negative charge (Fig. 17-14).

It follows from the definition of the gravitational field as force per unit mass that if we place a mass m in a gravitational field **g** the mass experiences a force $\mathbf{F} = m\mathbf{g}$. Thus a 10 kg mass at the surface of the earth, where $\mathbf{g} = 9.8$ N/kg directed toward the earth's center, will experience a force of magnitude 98 N in the direction of **g**. At a distance $r = 2R$ from the center of the earth, where $\mathbf{g} = 2.5$ N/kg directed inward, the same 10 kg mass would experience a force of magnitude 25 N in the direction of **g**.

Group of Point Source Charges

When the source of an electric field is a group of point charges ($q_1, q_2, \ldots, q_n$), the electric field is the **resultant** electric force per unit charge on a test charge q' placed at a field point; that is,

$$\mathbf{E} = \frac{\mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n}{q'}$$

$$= \frac{\mathbf{F}_1}{q'} + \frac{\mathbf{F}_2}{q'} + \cdots + \frac{\mathbf{F}_n}{q'}$$

Each term in this equation is the field that would be produced by one charge alone. So **the total electric field E is the vector sum of the fields produced by the individual charges:**

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots + \mathbf{E}_n \qquad (17\text{-}9)$$

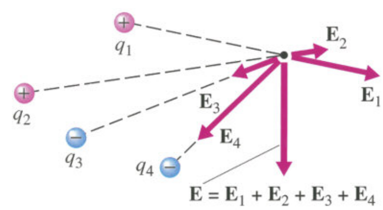


Fig. 17-16 The field produced by a group of point charges is the vector sum of the single charge fields.

Fig. 17-16 illustrates the calculation of the electric field. You first calculate the electric field of individual charges as before and then compute the **vector** sum of these fields.

EXAMPLE 6 The Electric Field of Two Charges

Point charges $q_1 = +5.00 \times 10^{-9}$ C and $q_2 = -5.00 \times 10^{-9}$ C are on the x-axis with respective coordinates $x = -5.00$ cm and $x = +5.00$ cm. Find the electric field at three points: point a at the origin; point b on the x-axis at $x = -10.0$ cm; and point c on the y-axis, 10.0 cm from each charge.

SOLUTION

At field point a, charges q_1 and q_2 produce fields $\mathbf{E}_1$ and $\mathbf{E}_2$, both directed to the right, away from q_1 and toward q_2 (Fig. 17-17). The fields have equal magnitudes, since the source charges have equal magnitudes and are equal distances away from a:

$$E_2 = E_1 = k\frac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N/C})\frac{5.00 \times 10^{-9}\text{ C}}{(5.00 \times 10^{-2}\text{ m})^2}$$
$$= 1.80 \times 10^4 \text{ N/C}$$

The total field $\mathbf{E}$ at point a is directed to the right and has magnitude

$$E = E_1 + E_2 = 2(1.80 \times 10^4 \text{ N/C})$$
$$= 3.60 \times 10^4 \text{ N/C}$$

At field point b, q_1 produces a field $\mathbf{E}_1$ to the left (away from q_1) and q_2 produces a field $\mathbf{E}_2$ to the right (toward q_2):

$$E_1 = k\frac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N/C})\frac{5.00 \times 10^{-9}\text{ C}}{(5.00 \times 10^{-2}\text{ m})^2} = 1.80 \times 10^4 \text{ N/C}$$

$$E_2 = k\frac{|q_2|}{r_2^2} = (8.99 \times 10^9 \text{ N/C})\frac{5.00 \times 10^{-9}\text{ C}}{(15.0 \times 10^{-2}\text{ m})^2} = 0.200 \times 10^4 \text{ N/C}$$

The total electric field is directed to the left and has magnitude

$$E = E_1 - E_2 = 1.60 \times 10^4 \text{ N/C}$$

At point c, $\mathbf{E}_1$ and $\mathbf{E}_2$ are directed as indicated in Fig. 17-17. The fields have equal magnitudes because the two source charges are equidistant from c:

$$E_2 = E_1 = k\frac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N/C})\frac{5.00 \times 10^{-9}\text{ C}}{(0.100\text{ m})^2}$$
$$= 4.50 \times 10^3 \text{ N/C}$$

The total field is found by vector addition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$E_x = E_{1x} + E_{2x} = (4500 \text{ N/C})(\cos 60.0°) + (4500 \text{ N/C})(\cos 60.0°)$$
$$= 4.50 \times 10^3 \text{ N/C}$$

The vertical components of $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel, and so the y component of $\mathbf{E}$ is 0. Thus the field is directed to the right and has a magnitude of 4.50×10^3 N/C.

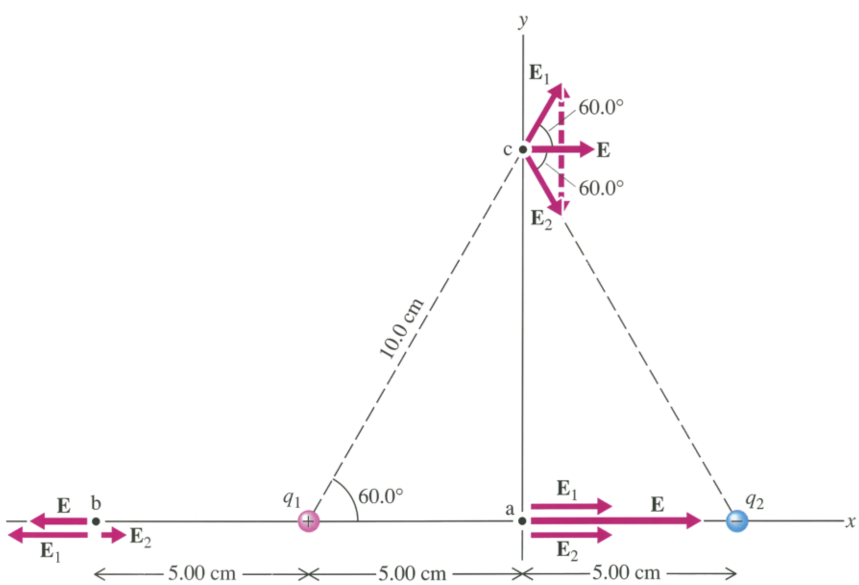


Fig. 17-17

EXAMPLE 7 Acceleration of an Electron in an Electric Field

Find the instantaneous acceleration of an electron placed at field point c in the previous example.

SOLUTION First we calculate the force on the electron, using Eq. 17-7 ($\mathbf{F} = q'\mathbf{E} = -e\mathbf{E}$). Since the charge is negative, the direction of the force is opposite the direction of the electric field. The field at c is directed to the right, and so the force on the electron is directed to the left. We find the magnitude of the force by multiplying the magnitude of the electron's charge times the magnitude of the field at c, found in the previous example to be 4.50×10^3 N/C:

$$F = |q'|E = eE$$
$$= (1.60 \times 10^{-19}\ \text{C})(4.50 \times 10^3\ \text{N/C})$$
$$= 7.20 \times 10^{-16}\ \text{N}$$

Since this is the only force acting on the electron, the acceleration of the electron, according to Newton's second law, is also directed to the left and has magnitude

$$a = \frac{F}{m} = \frac{7.20 \times 10^{-16}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}}$$
$$= 7.90 \times 10^{14}\ \text{m/s}^2$$

EXAMPLE 8 The Electric Field of Three Charges

Charges $q_1 = 1.0 \times 10^{-6}$ C, $q_2 = -1.0 \times 10^{-6}$ C, and $q_3 = 2.0 \times 10^{-6}$ C are located on the y-axis, with respective y coordinates 10 cm, 0, and −10 cm. Find the electric field at the field point having coordinates $x = 10$ cm, $y = 0$.

SOLUTION First we indicate in Fig. 17-18 the directions of $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$ produced by q_1, q_2, and q_3. Next we calculate the magnitude of the fields:

$$E_1 = k\frac{|q_1|}{r_1^2} = (9.0 \times 10^9\ \text{N-m}^2/\text{C}^2)\frac{1.0 \times 10^{-6}\ \text{C}}{(0.10\sqrt{2}\ \text{m})^2} = 4.5 \times 10^5\ \text{N/C}$$

$$E_2 = k\frac{|q_2|}{r_2^2} = (9.0 \times 10^9\ \text{N-m}^2/\text{C}^2)\frac{1.0 \times 10^{-6}\ \text{C}}{(0.10\ \text{m})^2} = 9.0 \times 10^5\ \text{N/C}$$

$$E_3 = k\frac{|q_3|}{r_3^2} = (9.0 \times 10^9\ \text{N-m}^2/\text{C}^2)\frac{2.0 \times 10^{-6}\ \text{C}}{(0.10\sqrt{2}\ \text{m})^2} = 9.0 \times 10^5\ \text{N/C}$$

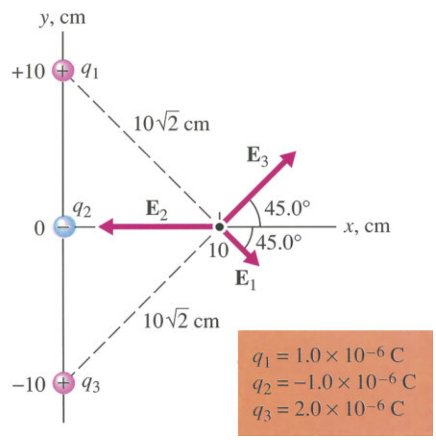


Fig. 17-18

Finally we calculate the vector sum, using the calculated magnitudes and the directions indicated in Fig. 17-18:

$$E_x = E_{1x} + E_{2x} + E_{3x}$$
$$= (4.5 \times 10^5\ \text{N/C})(\cos 45°) - 9.0 \times 10^5\ \text{N/C} + (9.0 \times 10^5\ \text{N/C})(\cos 45°)$$
$$= 5.5 \times 10^4\ \text{N/C}$$

$$E_y = E_{1y} + E_{2y} + E_{3y}$$
$$= -(4.5 \times 10^5\ \text{N/C})(\sin 45°) + 0 + (9.0 \times 10^5\ \text{N/C})(\sin 45°)$$
$$= 3.2 \times 10^5\ \text{N/C}$$

This is a vector of magnitude 3.2×10^5 N/C, directed 80° above the positive x-axis.

In the examples above we identified certain charges as sources of an electric field and then calculated the magnitude and direction of that field. You should realize that these source charges can also respond to a field. Thus each charge both is the source of an electric field and also experiences a force caused by the electric field of all other charges.

17-4 Fields Produced by Continuous Distributions of Charge

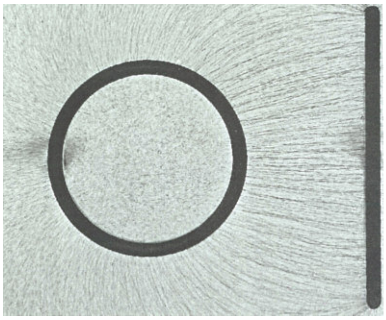


Fig. 17-19 The electric field produced by two charged conductors is made visible by a suspension in oil of bits of thread, which align with the field.

Often the source of an electric field is more than just a few point charges, and so the methods of the last section are not sufficient to compute the field. For example, a charged metal plate might have perhaps 10^{10} excess electrons on its surface. In computing the electric field produced by the plate, we certainly can't compute the fields of 10^{10} electrons, point by point. Instead we can think of the excess electrons as forming a continuous distribution of charge on the plate's surface and in so doing simplify the computation of the field. In this section we describe the electric fields produced by such charge distributions (Fig. 17-19).

The first step in evaluating the electric field produced at a field point P by a continuous distribution of charge is to divide the charge into tiny elements of charge Δq and then consider the field $\Delta\mathbf{E}$ produced by each such element at P (Fig. 17-20). Since Δq is so small, we can use the formula for a point charge to express the magnitude of the field $\Delta\mathbf{E}$ produced by Δq. Applying Eq. 17-8 $\left(E = k\frac{|q|}{r^2}\right)$ to our "point" charge Δq, we may write

$$|\Delta\mathbf{E}| = k\frac{|\Delta q|}{r^2} \qquad (17\text{-}10)$$

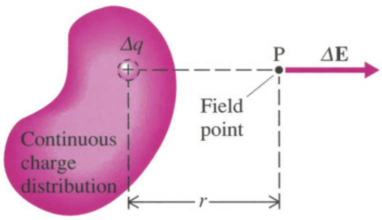


Fig. 17-20 Field $\Delta\mathbf{E}$ produced at a field point P by a small charge element Δq (assumed positive), part of a continuous charge distribution.

The field $\Delta\mathbf{E}$ is directed away from Δq if Δq is positive and toward Δq if Δq is negative. We find the total electric field at P by applying the superposition principle, that is, by summing the $\Delta\mathbf{E}$'s that were calculated using Eq. 17-10:

$$\mathbf{E} = \Sigma\,(\Delta\mathbf{E}) \qquad (17\text{-}11)$$

EXAMPLE 9 Electric Field of a Charged Ring

A thin ring is uniformly charged with a positive charge Q. A field point P is located on the axis of the ring at a distance r from the edge of the ring (Fig. 17-21). Derive an expression for the electric field at P.

SOLUTION Charge Q is uniformly spread over the ring, and since each element of charge Δq is the same distance r from the field point, each contributes a field of the same magnitude ΔE. Fig. 17-21 shows a pair of charge elements, Δq and $\Delta q'$, located on opposite sides of the ring, along with their respective fields, $\Delta\mathbf{E}$ and $\Delta\mathbf{E}'$, at field point P. The components perpendicular to the axis, $\Delta E_\perp$ and $\Delta E'_\perp$, cancel. This means that the only components contributing to the total field are those along the x-axis, ΔE_x and $\Delta E'_x$. Since the entire ring can be divided into other pairs of charges comparable to Δq and $\Delta q'$, all field components perpendicular to the x-axis cancel. So only components along the x-axis need to be summed to obtain the total electric field E_x:

$$E_x = \Sigma\ (\Delta E_x)$$

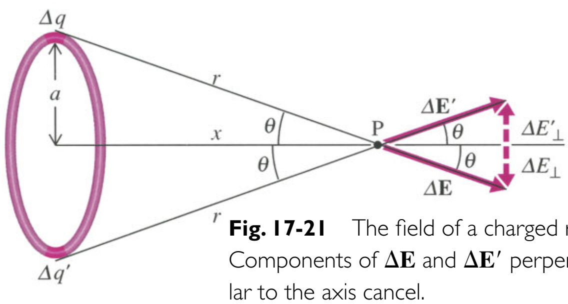


Fig. 17-21 The field of a charged ring. Components of $\Delta\mathbf{E}$ and $\Delta\mathbf{E}'$ perpendicular to the axis cancel.

Each of the n charge elements Δq contributes an equal component $\Delta E_x = \Delta E\cos\theta$, where ΔE is given by Eq. 17-10 $\left(\Delta E = k\frac{|\Delta q|}{r^2}\right)$.

Thus

$$E_x = \Sigma\left[k\frac{|\Delta q|}{r^2}(\cos\theta)\right] = nk\frac{\Delta q}{r^2}(\cos\theta)$$

Since $n\,\Delta q$ equals the total charge Q on the ring, we may express this result as

$$E_x = k\frac{Q}{r^2}(\cos\theta) \qquad (17\text{-}12)$$

Charge Density

In the preceding example, charge was distributed over a line. More often, however, we encounter problems in which charge is distributed over either a surface area or a volume. Such distributions lead to the definitions of surface charge density and volume charge density. **Surface charge density,** denoted by the Greek letter σ (sigma), is defined as **charge per unit area. Volume charge density,** denoted by the Greek letter ρ (rho), is defined as **charge per unit volume.**

$$\sigma = \frac{Q}{A} \tag{17-13}$$

$$\rho = \frac{Q}{V} \tag{17-14}$$

The units for σ are C/m^2, and the units for ρ are C/m^3.

EXAMPLE 10 Charge Density in a Uranium Nucleus

The uranium nucleus has a radius of 7.4×10^{-15} m and carries a charge of +92e. Find the charge density within the nucleus.

SOLUTION Since the nuclear charge is spread over the volume of the nucleus, we apply the definition of volume charge density (Eq. 17-14) and use the formula for the volume of a sphere of radius r ($V = \frac{4}{3}\pi r^3$).

$$\rho = \frac{Q}{V} = \frac{+92e}{\frac{4}{3}\pi r^3} = \frac{+92(1.6 \times 10^{-19}\text{ C})}{\frac{4}{3}\pi(7.4 \times 10^{-15}\text{ m})^3}$$

$$= 8.7 \times 10^{24}\text{ C/m}^3$$

Such a large charge density can occur within a nucleus because the large forces of electrical repulsion between the protons are balanced by attractive nuclear forces.

Uniformly Charged Infinite Plane

An important special case of a continuous charge distribution is the infinite, uniformly charged plane (Fig. 17-22a). The solution of a problem involving an infinite distribution of charge might seem to be an exercise of no physical significance. However, the solution to this problem is a good approximation to the field a *finite* plane of charge produces at field points that are close enough to the surface of the plane and far enough from the edges to make the plane "look" infinite (Fig. 17-22b). For such field points, the missing part of the infinite plane (from the edges of the actual plane out to infinity) will contribute a negligible amount to the total electric field. Thus the realistic problem of a finite plane of charge can be approximated by the problem of an infinite plane, which has a simple solution.

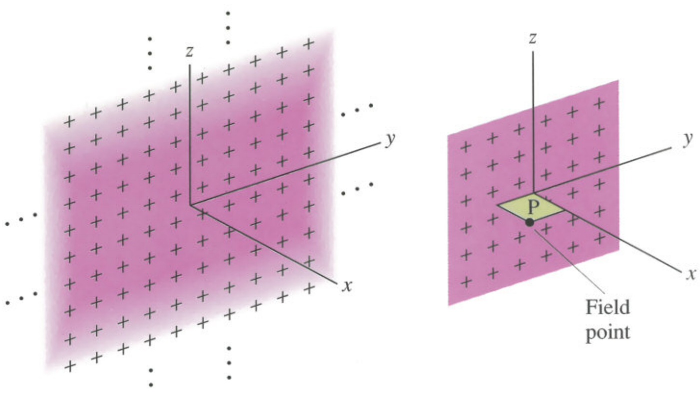


Fig. 17-22 **(a)** An infinite, uniformly charged plane. **(b)** A finite, uniformly charged plane. Point P is close enough to the plane and far enough from the edges that the plane looks infinite.

It is possible to derive the infinite plane's electric field by applying Eq. 17-11, but since this requires the use of integral calculus, we shall state the result here without proof. (However, a derivation based on Gauss's law is provided in Appendix B.)

The electric field of an infinite plane having a uniform surface charge density σ is uniform, is directed perpendicular to the plane, and has magnitude

$$E = 2\pi k|\sigma| \quad \text{(infinite plane; constant } \sigma) \qquad (17\text{-}15)$$

The field of a positively charged plane is directed away from the plane (Fig. 17-23a). The field of a negatively charged plane is directed toward the plane (Fig. 17-23b).

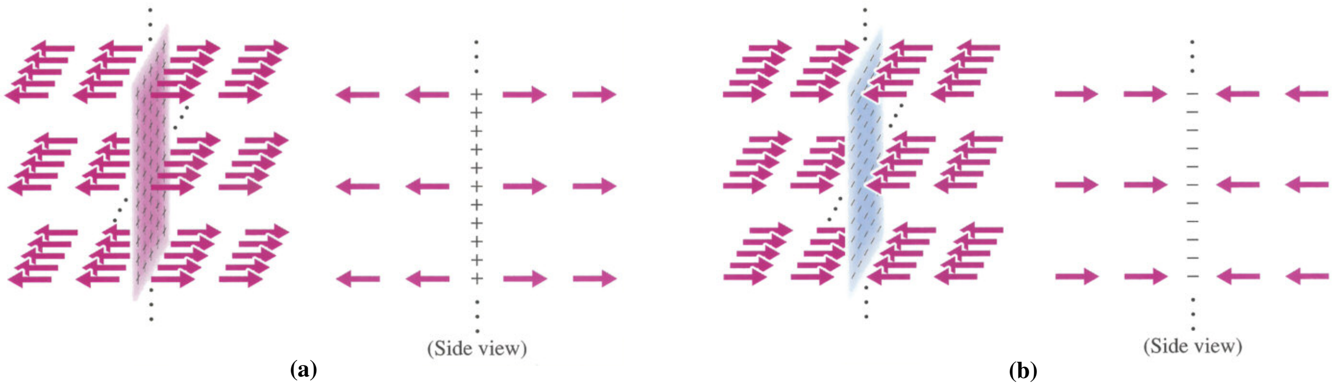


Fig. 17-23 **(a)** Electric field of a positively charged infinite plane. **(b)** Electric field of a negatively charged infinite plane.

EXAMPLE 11 Electric Field of a Charged Sheet of Photocopy Paper

Inside a photocopier, a sheet of copy paper with dimensions of 20 cm by 30 cm has 1.0×10^{11} electrons removed from one side, producing a uniform positive surface charge. The purpose of this positive charge is to attract the negatively charged black ink, or "toner," from the photocopier drum, where the image of the original is first formed; when the toner is attracted off the drum, the image is transferred onto the copy paper. Find the electric field at a point 1.0 cm from the surface of the paper and not too close to the edge.

SOLUTION The uniformly charged sheet has a surface charge density

$$\sigma = \frac{Q}{A} = \frac{(1.0 \times 10^{11})e}{A} = \frac{(1.0 \times 10^{11})(1.60 \times 10^{-19}\text{ C})}{(0.20\text{ m})(0.30\text{ m})}$$

$$= +2.7 \times 10^{-7}\text{ C/m}^2$$

This charge produces an electric field that is directed away from the sheet and has a magnitude given by Eq. 17-15:

$$E = 2\pi k|\sigma| = 2\pi(9.0 \times 10^9\text{ N-m}^2/\text{C}^2)(2.7 \times 10^{-7}\text{ C/m}^2)$$

$$= 1.5 \times 10^4\text{ N/C}$$

Isolated Conducting Plate

Suppose a net positive charge is placed on a large, isolated metal plate, say, by removal of some electrons through static charging. The remaining free electrons in the metal will arrange themselves in such a way that the net positive charge will be distributed equally over the surface of the plate, with equal charge on the two sides (Fig. 17-24). The electric field at any point is the vector sum of the fields produced by the sides, each of which has a positive charge density σ. Each side produces a field that is approximately the field of an infinite plane, that is, a field of magnitude $2\pi k\sigma$, directed away from the surface. To either side of the plate, the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ produced by the two surface charges are in the same direction, and thus the total field to either side is directed away from the plate and has a magnitude twice that produced by one surface:

$$E = 2(2\pi k\sigma) = 4\pi k\sigma \tag{17-16}$$

Inside the metal, the direction of $\mathbf{E}_1$ is opposite the direction of $\mathbf{E}_2$. Thus these fields cancel and the resultant field inside is zero:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \mathbf{0} \qquad \text{(inside conductor)} \tag{17-17}$$

Indeed, the static charge arranges itself on the surface of the plate in such a way as to produce zero field inside. If a nonzero field **E** were present inside, the free electrons in the metal would experience a force $\mathbf{F} = -e\mathbf{E}$ and would keep moving until the arrangement of charge produced zero field inside.

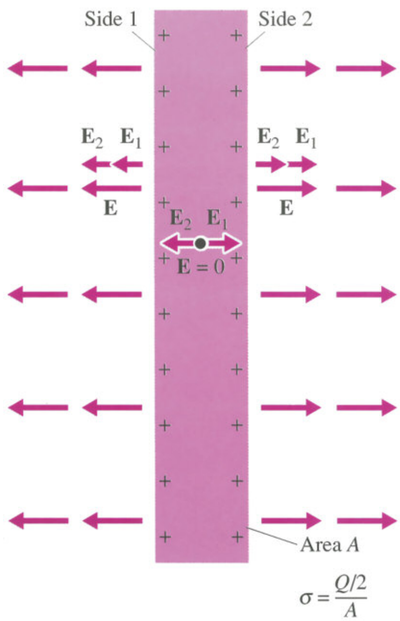


Fig. 17-24 Side view of an isolated conducting plate, with charge equally divided between the two sides. The two charged sides produce fields ($\mathbf{E}_1$ and $\mathbf{E}_2$) that cancel inside the plate but not outside.

Properties of Conductors

Several features of the charged metal plate are properties of charged conductors of *any* shape. The most important general properties of charged conductors are:

1. **The electric field inside a statically charged conductor always equals zero.** The free electrons in the conductor keep moving until the charge distribution produces zero electric field inside.
2. **Any net charge on a conductor is on the surface.** It is possible to prove this property by applying Gauss's law (Appendix B). Gauss's law shows why a net charge inside a conductor must always produce a nonzero field inside, and since there is no field inside, there can be no charge.
3. **The electric field within a cavity in a conductor is always zero,** if there is no charge inside the cavity. This property can be proved by use of Gauss's law and the fact that the electric force is a conservative force. This feature of conductors can be utilized to shield a region of space from electric fields produced by charges outside the region. All that is required is a closed conducting surface surrounding the region of space to be shielded. An automobile's metal body almost completely surrounds the automobile's interior with a conductor and hence provides good protection against the car being penetrated by electric fields, produced, for example, by an electrical storm (Fig. 17-25).
4. **The electric field just outside the surface of a conductor is perpendicular to the surface and has magnitude**

$$E = 4\pi k|\sigma| \tag{17-18}$$

where $|\sigma|$ is the magnitude of the surface charge density on the part of the conductor closest to the field point (Fig. 17-26). This property is a generalization of the result for a conducting plate and, like properties 2 and 3, can be derived from Gauss's law (Appendix B).

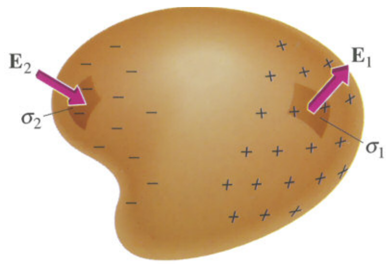


Fig. 17-26 Charge density varies over the surface of this conductor. The electric field just outside any part of the surface is proportional to the charge density there.

Fig. 17-25 The metal body of a car shields the interior from electric fields.

5. **The surface charge density on a conductor is greatest where the surface is least flat, especially at sharp points.** This property can be proved using the concept of electric potential (see Problem 56, Chapter 17). According to Eq. 17-18 ($E = 4\pi k|\sigma|$), the field is strongest where σ is greatest. Therefore the strongest fields around a charged conductor are located just outside sharp points, as indicated in Fig. 17-27. Lightning rods utilize this principle. These sharply pointed metal rods, connected to ground, are sometimes placed on tall buildings. They protect the buildings in two ways. First, they tend to prevent the occurrence of lightning in the immediate vicinity of the building, since the enhanced field near a rod's sharp point will tend to discharge a nearby thundercloud before it reaches the high charge concentrations necessary for a lightning discharge. Second, any lightning that does occur in the vicinity is likely to be through the rod to the ground, rather than through the building. The Empire State building, which is protected by a lightning-rod system, is struck by lightning every few weeks.

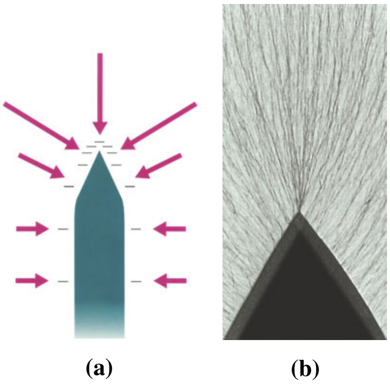


Fig. 17-27 **(a)** The electric field around a charged conductor is strongest near a sharp point. **(b)** Bits of thread suspended in oil surround a sharply pointed, charged conductor. Notice that the threads are concentrated near the point, indicating that the field is strongest there.

17-5 Field Lines

The direction of an electric field can be graphically represented by continuous lines called **field lines. The direction of the field at any point is the direction of the tangent to the field line at that point** (Fig. 17-28).

The magnitude of the electric field can also be indicated by field lines but not by their length, since the lines are continuous. Instead, **the spacing of the lines indicates the strength of the field.** Consider any surface perpendicular to the field lines (Fig. 17-29). The lines are drawn so that the magnitude of **E** is proportional to the number n of field lines per unit area through a surface of area $A_\perp$, perpendicular to the field lines:

$$E \propto \frac{n}{A_\perp} \tag{17-19}$$

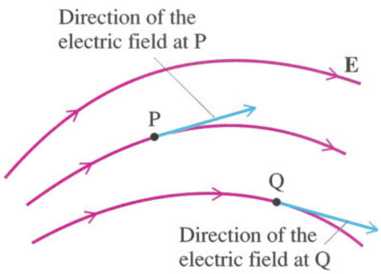


Fig. 17-28 Field lines.

Where the field lines are closer together, the number of lines per unit area is greater and so the electric field is stronger. Where the field lines are farther apart, the electric field is weaker (Fig. 17-29).

The field line representation is useful because it is often possible to draw field lines that are continuous through most regions of space. **Field lines begin and end only at points where there is electric charge.** This general property of field lines is proved in Appendix B on Gauss's law. However, it is easy to verify the continuity of field lines for two special cases: the uniformly charged, infinite plane and the point charge.

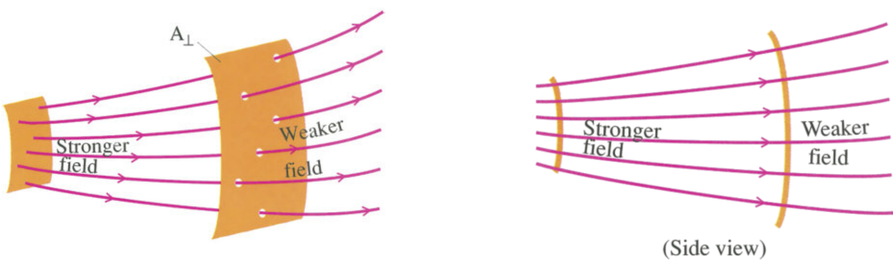


Fig. 17-29 The same number of field lines passes through two surfaces. The number of lines per unit surface area is greater for the smaller surface, indicating the field is greater there.

Field Lines for a Uniformly Charged, Infinite Plane

As described in the previous section, the field of a uniformly charged, infinite plane is uniform and perpendicular to the plane. The field line representation of such a field consists of continuous, equally spaced lines extending out in either direction from the plane and perpendicular to it (Fig. 17-30).

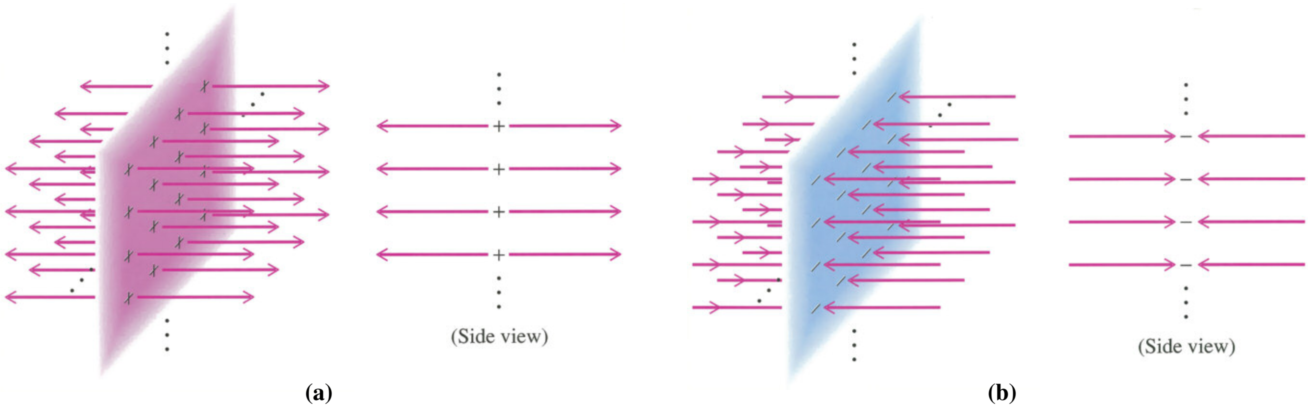


Fig. 17-30 Field lines for a uniformly charged infinite plane. **(a)** Positively charged plane. **(b)** Negatively charged plane.

According to Eq. 17-19, electric field strength is proportional to the number of field lines n per unit area $A_\perp$. Thus the number of field lines through a perpendicular surface is proportional to the product of field strength and surface area:

$$n \propto EA_\perp$$

Or we may introduce a constant of proportionality c and express this result as

$$n = cEA_\perp \tag{17-20}$$

The constant c is a scale factor, chosen so as to obtain the desired number of lines in a drawing.

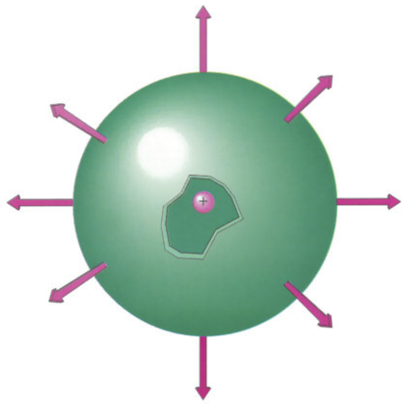


Fig. 17-31 This spherical surface is perpendicular to the electric field produced by a point charge at the center of the sphere.

Field Lines for a Single Point Charge

We shall apply Eq. 17-20 to the problem of representing by field lines the electric field of a positive point charge q. Since the field is directed radially outward from the charge, we choose as a perpendicular surface a spherical surface that has radius r and is centered on the charge (Fig. 17-31). We apply Eq. 17-20, using the expression for the field of a point charge q at a distance r and the equation for the surface area of a sphere of radius r:

$$n = cEA_\perp = c\left(k\frac{q}{r^2}\right)(4\pi r^2)$$

or

$$n = 4\pi kqc \tag{17-21}$$

Notice that this expression for n involves only constants. This means that the number of field lines passing through a sphere of *any* radius r is the same, independent of r. Thus we can represent the field by continuous field lines (Fig. 17-32). All the lines that originate at q pass through any spherical surface centered on q. The number of lines drawn from q will depend on the value chosen for the scale factor c.

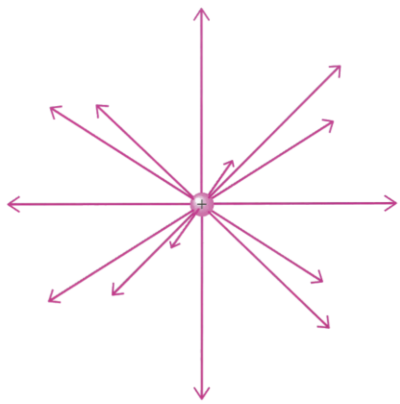


Fig. 17-32 Field lines of a point charge.

Field of a Uniformly Charged Sphere

A charged sphere in which charge is uniformly distributed throughout the volume of the sphere produces an electric field that, for field points outside the sphere, is the same as though the charge were concentrated at the center of the sphere; that is, the field outside the sphere is the field of a point charge.* This result, proved in Appendix B on Gauss's Law, means that the field lines outside a uniform sphere of charge look the same as for a point charge (Fig. 17-32).

Two-Dimensional Drawings of Field Lines

It is difficult to indicate a three-dimensional field line pattern by a perspective drawing. Therefore, we shall often use two-dimensional drawings of field lines. For example, Fig. 17-33 shows in two dimensions the field lines of an electric dipole, which consists of two point charges of opposite sign but equal magnitude. When viewing such figures, you should keep in mind that the actual pattern of field lines is three-dimensional.

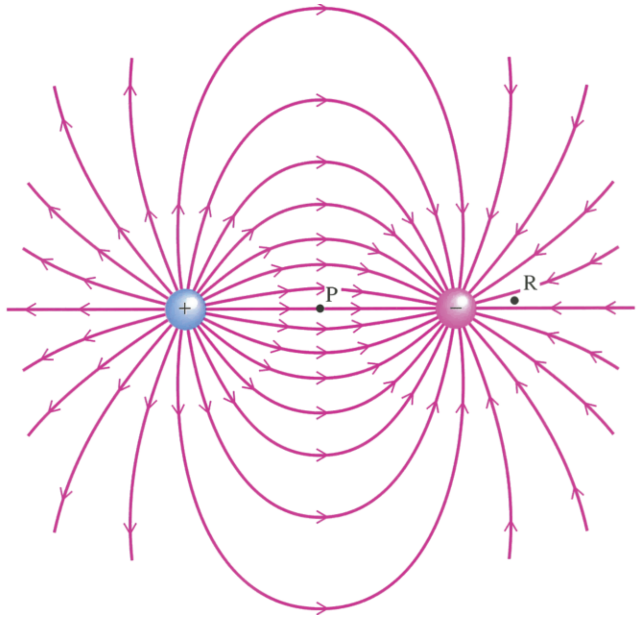


Fig. 17-33 Field lines of a dipole.

EXAMPLE 12 Finding the Strength of a Field From the Number of Field Lines

Suppose that two small surfaces of equal area are drawn in Fig. 17-33—one a plane surface passing through point P and perpendicular to field lines there and the other a spherical surface centered on the negative charge and passing through point R. Suppose that 9 field lines pass through the plane surface and 72 pass through the spherical surface. If the strength of the electric field at P is 200 N/C, what is it at R?

SOLUTION Since field strength is proportional to the number of lines per unit area and the two surface areas are equal, the field strength is proportional to the number of lines. Thus the field strength at R is

$$E_R = \tfrac{72}{9} E_P = 8(200 \text{ N/C})$$
$$= 1600 \text{ N/C}$$

*Similarly, the earth's gravitational field is the same for points outside the earth as though the earth's mass were concentrated at its center. (See Chapter 6, Fig. 6-6.)

CHAPTER 17 SUMMARY

Electric charge q, measured in coulombs (C), may be either positive or negative. Charges of like sign repel one another; charges of opposite sign attract one another. Uncharged matter consists of equal numbers of protons ($q = +e$) and electrons ($q = -e$), where

$$e = 1.60 \times 10^{-19}\ \mathrm{C}$$

Electrons may be transferred to or from a body, giving it a net charge proportional to the number n of electrons transferred:

$$q = \pm ne$$

The magnitude of the mutual attractive or repulsive force between point charges is proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance r between them, according to Coulomb's law:

$$F = k\frac{|q||q'|}{r^2}$$

where $$k = 8.99 \times 10^9\ \mathrm{N\text{-}m^2/C^2}$$

The superposition principle states that the resultant force produced by a number of charges on a charge q is found by first calculating the magnitudes and directions of the Coulomb forces each charge exerts on q and then finding the vector sum of these forces.

The electric field $\mathbf{E}$ is a vector quantity produced by a distribution of charge in the region of space surrounding that charge. A point in space where the field is evaluated is called a "field point."

A charge q' in an electric field experiences a force:

$$\mathbf{F} = q'\mathbf{E}$$

where $\mathbf{E}$ is the field at the location of q'; $\mathbf{E}$ is produced by other charges.

The field produced by a single point charge q is directed away from q if q is positive and toward q if q is negative. The magnitude of the field of a point charge at a distance r away from the charge is

$$E = k\frac{|q|}{r^2}$$

The field produced by a distribution of charge is the vector sum of the fields produced by each individual charge:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots + \mathbf{E}_n$$

For a continuous distribution of charge, we find the electric field by dividing the charge into tiny elements of charge Δq, calculating the field $\Delta\mathbf{E}$ of each element Δq as though it were a point charge, and then applying the superposition principle to find the total field:

$$\mathbf{E} = \Sigma(\Delta\mathbf{E})$$

where $$|\Delta\mathbf{E}| = k\frac{|\Delta q|}{r^2}$$

Surface charge density σ is defined as charge per unit area, and volume charge density ρ is defined as charge per unit volume:

$$\sigma = \frac{Q}{A}$$

$$\rho = \frac{Q}{V}$$

An infinite plane having a uniform charge density σ produces a uniform field, directed perpendicular to the plane—away from the plane if $\sigma > 0$ and toward the plane if $\sigma < 0$. The magnitude of the field is

$$E = 2\pi k|\sigma|$$

Both the electric field and the net charge inside a statically charged conductor equal zero. Any net charge on a conductor is on its surface, producing a surface charge density σ. The field at a field point just outside the conductor is perpendicular to the conductor's surface and has a magnitude determined by σ on the part of the surface near the field point, according to the equation

$$E = 4\pi k|\sigma|$$

Field lines are directed lines used to represent an electric field. They are continuous at most points in space and are often curved. Field lines terminate only at points in space where there is electric charge, beginning only on positive charges and ending only on negative charges. The tangent to a field line at any point indicates the direction of the field at that point. The magnitude of the field is indicated by the spacing of the lines—the number of lines per cross-sectional area is proportional to the strength of the field; thus the field is strongest where the lines are closest.

Questions

1 Is the electric force between an electron and a proton attractive or repulsive?

2 A positron is the antiparticle of an electron, which means that the positron has the same mass as the electron but it has a positive charge +e. Will a proton attract or repel a positron?

3 Physicists now know that protons and other particles previously thought to be indivisible are in fact made up of smaller units called "quarks." Protons consist of three quarks. There are various kinds of quarks, but all have electric charge of either $\pm\frac{2}{3}$e or $\pm\frac{1}{3}$e.

(a) If one of the quarks in a proton has a charge of $+\frac{2}{3}$e, what are the charges of the other two quarks?

(b) A neutron is an uncharged particle that, together with the proton, makes up most of the mass of an atom. Like the proton, the neutron consists of three quarks, one of which has an electric charge of $+\frac{2}{3}$e. What are the charges of the other two quarks?

4 The magnitude of the electric force between two point charges is initially 180 N. What will the force be if the distance between the two charges is (a) doubled; (b) tripled; (c) halved?

5 The water molecule has an electric dipole moment. This means that one end of the molecule is positively charged and the other end is negatively charged. These dipole moments result in electric forces between the molecules. Which of the pairs of dipoles shown in Fig. 17-34 experiences a net attractive force?

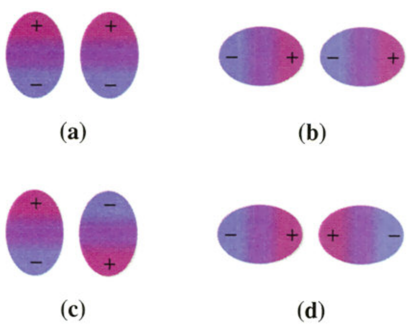


Fig. 17-34

6 A small positively charged object is brought close to one end of a long metal rod that is electrically insulated and initially uncharged. The object does not touch the rod. Does the rod exert a force on the object? Explain.

7 Two initially uncharged metal spheres are connected by a copper wire. A positively charged object is placed near one of the spheres but not touching it. What can you do to cause the two spheres to retain a charge even after the object is moved away? This process of charging without contact is called "charging by induction."

8 Fig. 17-35 shows two protons and one electron. What is the direction of the electric field at R?

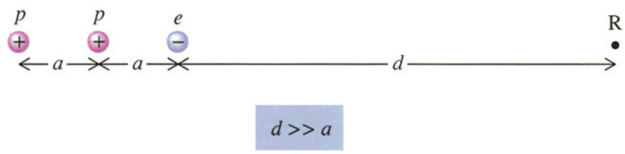


Fig. 17-35

9 All four charges in Fig. 17-36 have the same magnitude. At point P the electric field produced by these charges is directed toward the left. Charge q_1 is negative. What are the signs of q_2, q_3, and q_4?

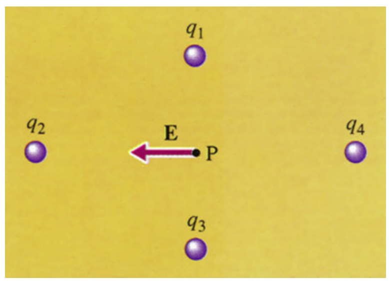


Fig. 17-36

10 Small pieces of paper are near a comb that carries a static charge. Although each piece of paper has no net charge, the comb's field causes the paper to become polarized, as shown in Fig. 17-37. All the positive charge in a piece of paper is pushed down in the direction of **E**, and all the negative charge is pushed up. By considering the magnitude and direction of the forces on the charges at the two ends of each piece of paper, determine whether this nonuniform field exerts a net force on the paper and, if so, in what direction.

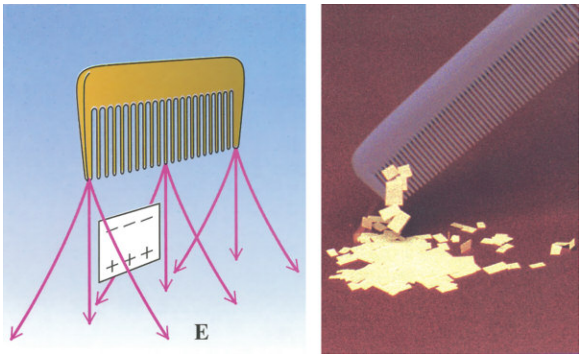


Fig. 17-37

11 In the "Star Trek" television series, prisoners on the starship *Enterprise* were confined to their cabins by means of an invisible force field in an open doorway (Fig. 17-38). When they attempted to pass through the doorway, they received a painful shock. Suppose the force field is just a strong electric field. A prisoner reasons that since his body is uncharged an electric field should not bother him. What's wrong with that reasoning?

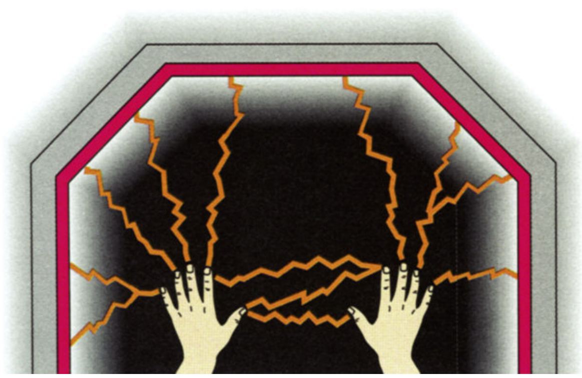


Fig. 17-38

12 Which of the sketches in Fig. 17-39, if any, can represent electric field lines?

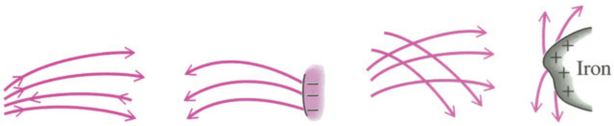


Fig. 17-39

13 Two uniformly charged infinite planes are shown in Fig. 17-40. If a positively charged balloon of negligible weight is attached by a string to point P, what will the direction of the string be when the balloon comes to rest?

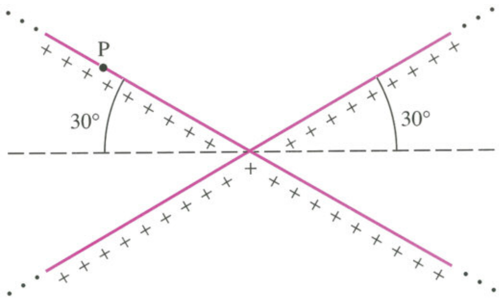


Fig. 17-40

14 If no other charge distribution is nearby, is it possible for the right side of a flat conducting plate to be charged while the left side remains uncharged? Explain.

15 Fig. 17-41 shows electric field lines around a conductor. (a) At which of the four points is the field strongest? (b) At which point is there negative charge?

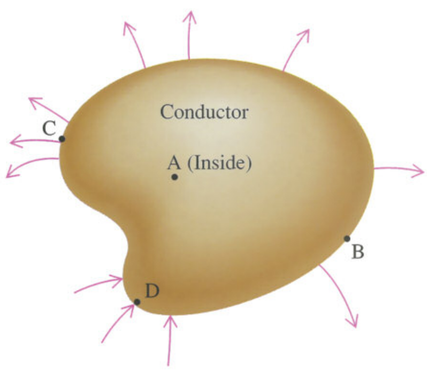


Fig. 17-41

16 Suppose you are driving through a thunderstorm. Would it be safer to be (a) inside a car with a metal body; (b) inside a fiberglass car; (c) outside your car?

17 Suppose you are in a cave, deep within the earth. Are you safe from electrical storms?

18 Suppose a positive charge of $+20e$ is moved through a small opening in a hollow metal sphere and placed at a point P, which is connected to the inside of the sphere by means of a metal wire (Fig. 17-42).

(a) What is the final charge distribution, and how is it achieved?

(b) Is there any limit to the amount of charge that can be transferred in this way, if the sphere is in a vacuum, so that there is no way for the charge to leak off ?

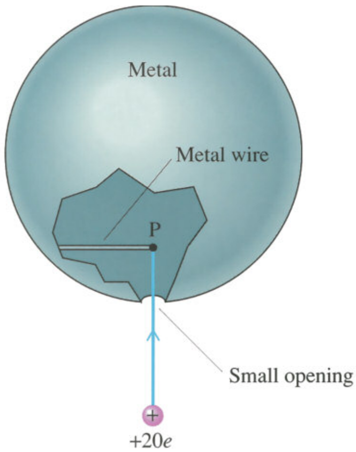


Fig. 17-42

19 A Van de Graaff generator operates on the principle described in the preceding question. Positive charges are supplied to the bottom end of a conveyor belt through frictional contact (Fig. 17-43a). The belt then carries the positive charge to the inside of a metal sphere, where the charge is transferred to the sphere's outer surface.

(a) Why must an upward force be applied to the positively charged side of the belt to lift it?

(b) When the girl in Fig. 17-43b touches the metallic surface, her body becomes an extension of the conductor, causing her skin to become positively charged and her hair to stand on end because of repulsion of like charges. Which of the following forces acts on the end of a hair, balancing the electric force of repulsion: attractive electric force, normal force, tension in the hair, or weight of the hair?

20 Determine the magnitude and sign of q_2 in Fig. 17-44.

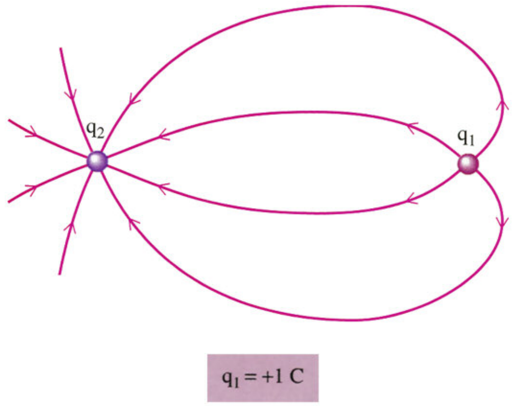
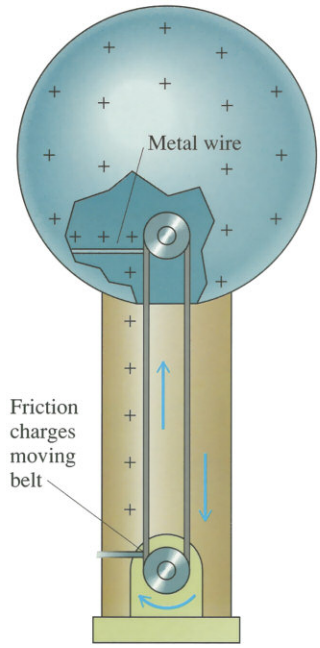


Fig. 17-44



(a)

(b)

Fig. 17-43

21 St. Elmo's fire is a glowing of the air that occurs under certain conditions around a charged object discharging into the atmosphere. This kind of discharge, also called a "corona discharge," is sometimes observed around the wings of an airplane in flight. Are you more likely to see St. Elmo's fire near the rounded front edge of the airplane's wing or near the sharp back edge?

Answers to Odd-Numbered Questions

1 attractive; **3** (a) $+\frac{2}{3}$e, $-\frac{1}{3}$e; (b) $-\frac{1}{3}$e, $-\frac{1}{3}$e; **5** b, c; **7** disconnect the wire; **9** −, −, +; **11** Although the body has no net charge, there are certainly charges within it; **13** vertical; **15** (a) C; (b) D; **17** yes; **19** (a) to balance the force of the downward-directed field acting on the belt's charges as a result of the charges already on the sphere (b) tension in the hair; **21** the sharp back edge

Problems (listed by section)

17-2 Coulomb's Law

1 A copper wire 90.0 cm long and 1.00 mm in diameter has a mass of 6.35 g.
(a) Find the number of electrons in the wire. (Copper has an atomic number of 29; that is, there are 29 protons in the copper atom. Copper's atomic mass is 63.5.)
(b) There is one free electron per atom in copper. Find the number of free electrons in the wire.

2 Donna and John have masses of 50.0 kg and 80.0 kg respectively.
(a) How many protons are there in each person? (Protons make up roughly 55% of the mass of the human body.)
(b) How many electrons are in each person?
(c) Suppose John and Donna stand 5.00 m apart. Calculate the force exerted on John's protons by (1) Donna's protons and (2) her electrons.
(d) What is the resultant force on his protons?

★ **3** The structure of a sodium chloride (table salt) crystal is shown in Fig. 17-45. Each sodium ion Na^+ has a charge +e and is adjacent to a chloride ion Cl^-, which has a charge −e. The electric force of attraction between sodium ions and chlorine ions holds the crystal together.
(a) What is the magnitude of the force between adjacent sodium and chlorine ions, 2.82×10^{-10} m apart?
(b) What is the resultant force on any ion in the crystal?
(c) Suppose you attempt to break a cubic salt crystal, 1.00 mm on a side, by applying forces **F** and **−F** perpendicular to opposite sides of the cube, trying to pull it apart. How great would F have to be to overcome the attractive forces of all the ions in a 1.00 mm^2 plane of the crystal?

★ **4** There are extremely large electrical forces of repulsion between the protons in the nucleus of an atom. However, these forces are normally not as great as the "strong force," which is the force that binds all the protons and neutrons in the nucleus together. The strong force has a very short range—on the order of 2×10^{-15} m. When protons or neutrons are separated by a distance greater than this, the strong force does not act. Thus, if for some reason a nucleus splits in two, or "fissions," each fragment can experience an electrical repulsive force without any other force to balance it. Suppose a uranium nucleus (92 protons) splits into two nuclei having 46 protons each.
(a) Calculate the repulsive force between these nuclei just after the split, when they are 10^{-14} m apart.
(b) Suppose that all the nuclei in one cross section of a 1 mm^3 cube (about 1.3×10^{11} nuclei) simultaneously split and experience a force perpendicular to the cross section. What would be the total force splitting the cube apart?

5 Find the ratio of the magnitudes of the electrical and the gravitational forces acting between a proton and an electron separated by an arbitrary distance d.

6 Find the force on a negative charge that is placed midway between two equal positive charges. All charges have the same magnitude.

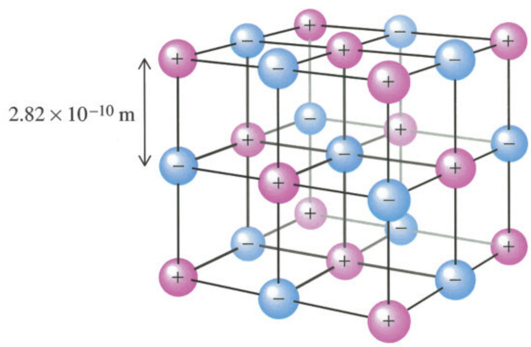


Fig. 17-45

★**7** Hanging from threads are two charged balls made of pith (a very light, spongy material that comes from inside the stems of certain plants). The balls each have a mass of 0.100 g and a charge of the same magnitude q and are attracted toward each other, as shown in Fig. 17-46.
(a) What is the magnitude of the electric force?
(b) What is the magnitude of q?
(c) How many electrons have been transferred to or from each ball?

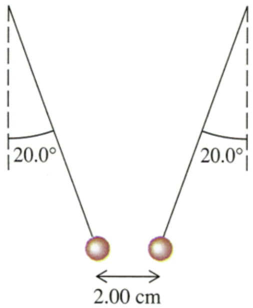
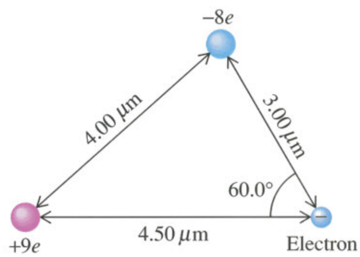


Fig. 17-46

8 Three charges, $q_1 = +2.00 \times 10^{-9}$ C, $q_2 = -3.00 \times 10^{-9}$C, and $q_3 = +1.00 \times 10^{-9}$C, are located on the x-axis at $x_1 = 0$, $x_2 = 10.0$ cm, and $x_3 = 20.0$ cm. Find the resultant force on q_3.

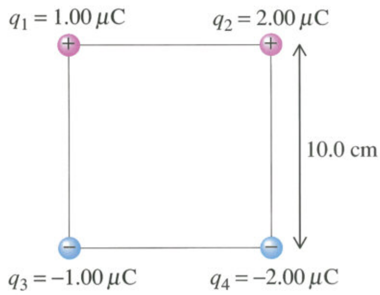
9 An electron is near a positive ion of charge +9e and a negative ion of charge −8e (Fig. 17-47).
(a) Find the magnitude and direction of the resultant force on the electron.
(b) Find the magnitude and direction of the electron's instantaneous acceleration.

Fig. 17-47



★**10** (a) In Fig. 17-48, what are the magnitude and direction of the resultant force on q_1?
(b) What is the resultant force on the center of mass of the four charges?

Fig. 17-48



11 In a Cartesian coordinate system, the charge $q_1 = -2.00 \times 10^{-4}$ C is at the origin, the charge $q_2 = 1.00 \times 10^{-3}$ C has coordinates $x = 10.0$ m, $y = 0$, and the charge $q_3 = -1.00 \times 10^{-4}$ C has coordinates $x = 0$, $y = -5.00$ m. Find the magnitude and direction of the resultant force on q_1.

★**12** Given two point charges q_1 and $q_2 = 4q_1$, find the position of a third charge q_3 relative to the other two charges, such that the resultant force on q_3 is zero.

17-3 The Electric Field

13 A charge q_1 on the y-axis produces a field of magnitude 3 N/C at the origin, and a charge q_2 on the x-axis produces a field of magnitude 4 N/C at the origin. What is the magnitude of the total field at the origin?

14 Fig. 17-49 shows fields $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, and $\mathbf{E}_4$ at point P, produced respectively by charges q_1, q_2, q_3, and q_4. What is the sign of each charge?

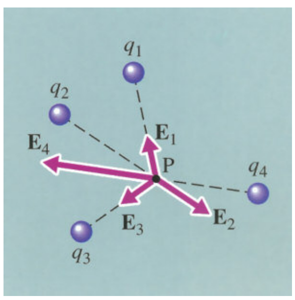


Fig. 17-49

15 Fig. 17-50 shows positive charges q_1 and q_2. At which of the five points, A, B, C, D, or G, could you place a negative charge q_3 of the right magnitude so that the field at point P is the field produced by q_1 alone?

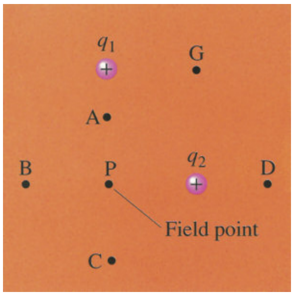


Fig. 17-50

16 In Fig. 17-51 two electrons are the same distance from a field point P. At which of the points A, B, C, or D could a proton be placed so that the electric field at P is zero?

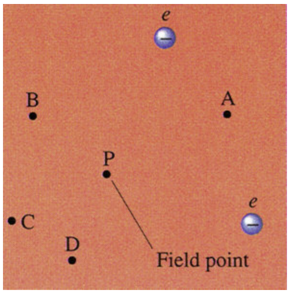


Fig. 17-51

17 Find the electric field at P in Fig. 17-52.

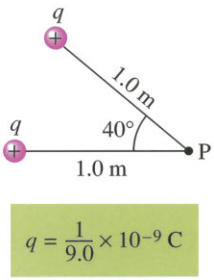


Fig. 17-52

18 At what distance from a proton does its electric field have a magnitude of 1 N/C?

19 Points A, B, and C are at the vertices of an equilateral triangle. A certain positive charge q placed at A produces an electric field of magnitude 100 N/C at C. Suppose a second, identical charge is placed at B. What is the magnitude of the new electric field at C?

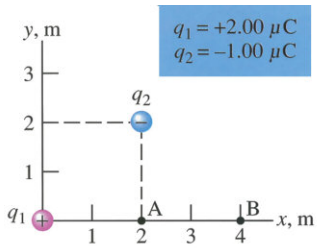
★ **20** Two point charges q_1 and q_2 are separated by 20.0 cm. The electric field at their midpoint is 600 N/C, directed away from q_1, which is $+1.00 \times 10^{-9}$ C. Find q_2.

21 A 2.00×10^{-9} C charge has coordinates $x = 0$, $y = -2.00$; a 3.00×10^{-9} C charge has coordinates $x = 3.00$, $y = 0$; and a -5.00×10^{-9} C charge has coordinates $x = 3.00$, $y = 4.00$, where all distances are in cm. Determine magnitude and direction for (a) the electric field at the origin and (b) the instantaneous acceleration of a proton placed at the origin.

22 Point charges of $+1.00 \times 10^{-9}$ C, $+1.00 \times 10^{-9}$ C, and -2.00×10^{-9} C are placed at the vertices of an equilateral triangle. Find the magnitude of the electric field at the center of the triangle, which is 10.0 cm from each vertex.

23 Find the x and y components of the electric field produced by q_1 and q_2 in Fig. 17-53 at (a) point A; (b) point B.

Fig. 17-53



24 A Ping-Pong ball that has a mass of 2.40 g is charged when 1.00×10^{11} electrons are added. The ball is stationary in an electric field. Find the magnitude and direction of the field.

★ **25** A small Styrofoam ball weighing 1.00×10^{-2} N is supported by a thread in a horizontal electric field of magnitude 1.00×10^{6} N/C. The thread makes an angle of 45.0° with the vertical. Find the magnitude of the charge on the ball.

26 Millikan measured the electron's charge by observing tiny charged oil drops in an electric field. Each drop had a charge imbalance of only a few electrons. The strength of the electric field was adjusted so that the electric and gravitational forces on a drop would balance and the drop would be suspended in air. In this way the charge on the drop could be calculated. The charge was always found to be a small multiple of 1.60×10^{-19} C. Find the charge on an oil drop weighing 1.00×10^{-14} N and suspended in a downward field of magnitude 2.08×10^{4} N/C.

★ **27** A beam of electrons is shot into a uniform downward electric field of magnitude 1.00×10^3 N/C. The electrons have an initial velocity of 1.00×10^7 m/s, directed horizontally. The field acts over a small region, 5.00 cm in the horizontal direction.
(a) Find the magnitude and direction of the electric force exerted on each electron.
(b) How does the gravitational force on an electron compare with the electric force?
(c) How far has each electron moved in the vertical direction by the time it has emerged from the field?
(d) What is the electron's vertical component of velocity as it emerges from the field?
(e) The electrons move an additional 20.0 cm after leaving the field. Find the total vertical distance that they have been deflected by the field.

28 During a thunderstorm the electric field at a certain point in the earth's atmosphere is 1.00×10^5 N/C, directed upward. Find the acceleration of a small piece of ice of mass 1.00×10^{-4} g, carrying a charge of 1.00×10^{-11} C.

17-4 Continuous Charge Distributions

29 The earth's surface has a charge density of about -1.0×10^{-9} C/m^2. Find the total charge on the surface.

★ **30** The earth's surface charge density (-1.0×10^{-9} C/m^2) is roughly balanced by a net positive charge in the lower 10 km of the earth's atmosphere. What is the average volume charge density of this atmospheric charge?

31 A thin gold ring of radius 1.00 cm carries a uniform charge per unit length of 1.00×10^{-14} C/m. Find the electric field on the axis of the ring 1.00 cm from the center.

★ **32** A 20.0 cm by 30.0 cm sheet of paper has a uniform surface charge density of 5.00×10^{-8} C/m^2. Find the electric field at a distance from the paper's surface of (a) 1.00 mm; (b) 1.00 cm; (c) 3.00 cm; (d) 5.00 m. None of the field points is close to the paper's edge.

★ **33** A rectangular slab of dimensions 1.00 m × 1.00 m × 10.0 cm has a uniform charge distribution of 1.00×10^{-7} C spread throughout its volume. Find the electric field at a point just outside the charge distribution, close to the center of one of the large faces of the slab.

★ **34** The earth is a good electrical conductor and, during periods of clear weather, has a downward-directed electric field of about 100 N/C at low altitudes. Find the charge density on the earth's surface when its field has this value.

35 Two large parallel planes each carry a uniform distribution of charge of the same magnitude σ but of opposite signs (Fig. 17-54). Find the electric field at points a, b, and c.

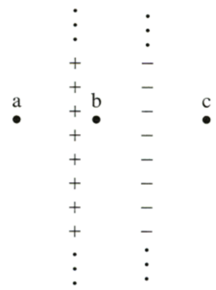


Fig. 17-54

36 Two large flat dielectric surfaces are parallel to each other and carry uniform charge densities (Fig. 17-55). Find the electric field at points a, b, and c.

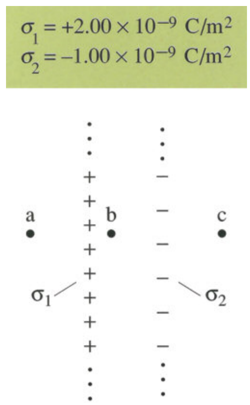


Fig. 17-55

37 Fig. 17-56 shows two large flat surfaces that have uniform charge densities. Find the electric field at points a and b.

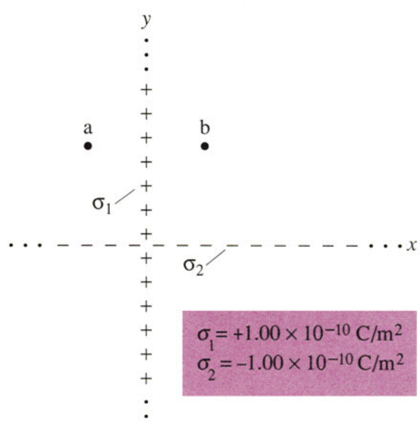


Fig. 17-56

38 A point charge q is near a uniformly charged, large flat surface of a dielectric (Fig. 17-57). Find the electric field at P.

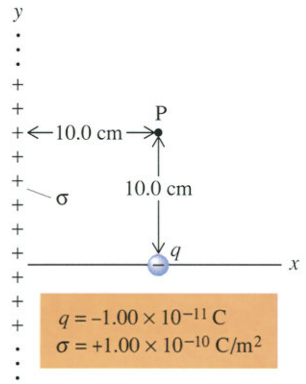


Fig. 17-57

17-5 Field Lines

39 Fig. 17-58 shows the field that results when a conducting sphere is placed in a uniform external field.
(a) Find the field at P, inside the conductor.
(b) At which of the points R, S, or T is the field weakest, and at which of these points is it strongest?
(c) Find the charge density on the surface of the conductor near R if the field at that point has magnitude 1.00×10^5 N/C.

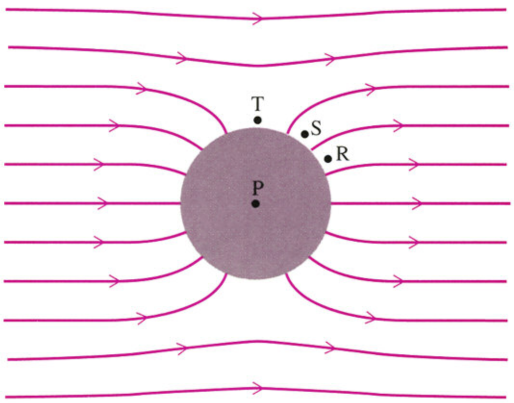


Fig. 17-58

40 A coaxial cable consists of an inner cylindrical conductor of radius 1.00 mm inside a hollow cylindrical conductor of radius 3.00 mm. Charges on the surfaces of these conductors produce the field lines shown in Fig. 17-59. The field lines are equally spaced along the axis of the cable. If the magnitude of the field just outside the inner conductor is 600 N/C, what is the magnitude of the field just inside the outer conductor?

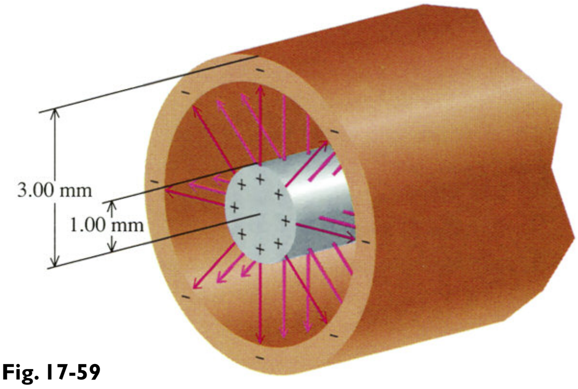


Fig. 17-59

Additional Problems

★ **41** Charge $q_1 = 1.00 \times 10^{-9}$ C is at the origin and charge $q_2 = 9.00 \times 10^{-9}$ C has coordinates $x = 40.0$ cm, $y = 0$. Find the coordinates of a field point where $\mathbf{E} = \mathbf{0}$.

★ **42** An aluminum sphere of radius 1.00 m carries a charge of -1.00×10^{-4} C. The sphere is isolated except for a particle of mass 1.00×10^{-15} kg and charge 1.00×10^{-17} C, which orbits the sphere in a circular orbit of radius 3.00 m. Find the period of the orbit.

★★ **43** Electric charge on the earth's surface and in the earth's atmosphere produces an electric field, which on a clear day (no thunderstorms) is directed vertically downward and has a magnitude of about 100 N/C just above the earth's surface. Very small negative particles can be supported by the field. For larger particles, the charge that must be carried may be so great that the field around the charge causes the surrounding air to conduct the charge away. This occurs at fields of about 3.0×10^6 N/C in dry air. Find the radius of the largest water drop that could be supported in the earth's electrostatic field. Assume that the drop is spherical.

Fig. 17-60 Above this field of wheat is an invisible atmospheric electric field. Even though you can't see it, the electric field is just as real as the wheat field.

★★ **44** Find the radius of the largest (spherical) water drop whose weight could be supported by any electrostatic field in dry air. The air becomes conducting when the *total* field at any point exceeds 3.0×10^6 N/C.

45 An electron is initially at rest just outside a large copper surface that has a charge density of 1.00×10^{-6} C/m^2. How far has the electron moved in 1.00×10^{-9} s if the field is uniform over this region?

46 A thundercloud contains a large concentration of charged particles: ionized molecules, charged drops of water, bits of ice, and specks of dust. There is a concentration of positive charge in the upper part of the cloud and of negative charge in the lower part.* Suppose that the charge distribution in a certain cloud can be approximated by two uniform spheres of charge +100 C and −100 C, centered at points P and Q (Fig. 17-61). Find the magnitude and direction of the electric field (a) at P and (b) at the location of an airplane 1.00 km directly above P.

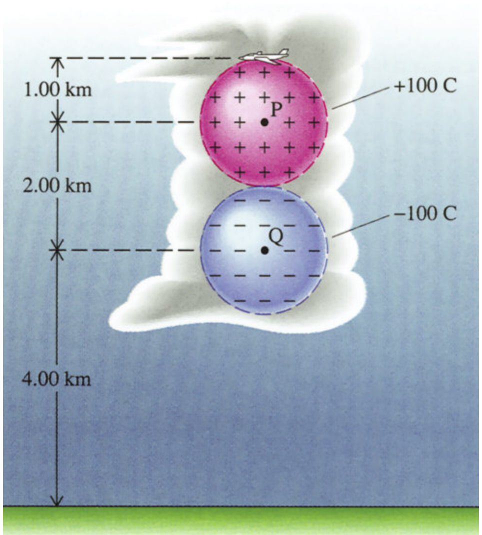


Fig. 17-61

*Negative charges are periodically transferred to the ground in the form of lightning strokes. Worldwide electrical storms throughout the day maintain the negative charge density on the earth's surface. This sporadic downward flow is balanced by a slow upward flow of negative charge during clear weather.

★ **47** A point charge q is placed midway between two identical positive point charges of magnitude 4.00×10^{-9} C. The resultant force on *each* of the three charges is zero. Find q.

48 The field just outside a point on the surface of a copper wire of radius 1.00 mm has magnitude 1.00×10^4 N/C.

(a) Find the magnitude of the surface charge density at that point on the wire.

(b) If the surface charge density is uniform around the circumference of the wire, how much charge is on a 1.00 m length of the wire?

★★ **49** A ring of radius 20.0 cm has uniform charge density of 1.00×10^{-6} C/cm. A 1.00 cm section of the ring on the right side is removed. What are the magnitude and direction of the electric field at the center of the ring?

★★ **50** Two point charges 0.600 m apart experience a repulsive force of 0.400 N. The sum of the two charges equals 1.00×10^{-5} C. Find the values of the two charges.